Relative and Absolute [mln59]

Lorentz invariant:

Consider frames S and S' in relative motion with velocity v.
A clock at rest in S signals a proper time interval $\Delta\tau$.

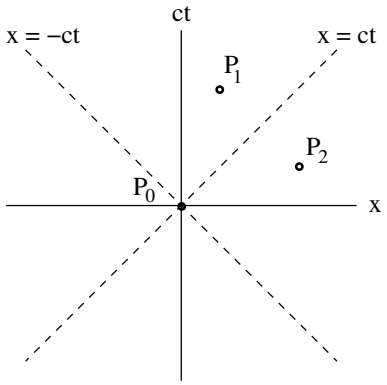
Time interval measured in S': $\Delta t' = \dfrac{\Delta\tau}{\sqrt{1 - v^2/c^2}}$.

Displacement of clock measured in S': $\Delta x' = -v\Delta t' = -\dfrac{v\Delta\tau}{\sqrt{1 - v^2/c^2}}$.

$$\Rightarrow\ (c\Delta t')^2 - (\Delta x')^2 = \frac{(c\Delta\tau)^2}{1 - v^2/c^2} - \frac{(v^2/c^2)(c\Delta\tau)^2}{1 - v^2/c^2} = (c\Delta\tau)^2.$$

Invariant spacetime distance: $(\Delta s)^2 \doteq (c\Delta t)^2 - (\Delta x)^2$.

Light cone:



Events P_0 and P_1 have $(\Delta s)^2 > 0$:

- time-like relation,
- causally related,
- at the same position in some frame.

Events P_0 and P_2 have $(\Delta s)^2 < 0$:

- space-like relation,
- not causally related,
- simultaneous in some frame.