Skater's Paradox [mln61]

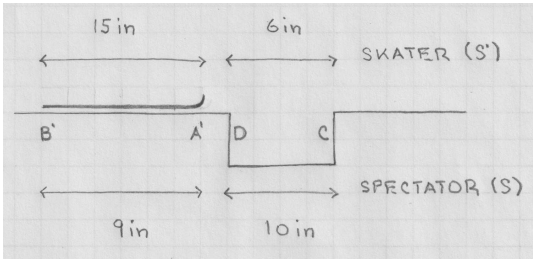
A skater with blades of proper length $\ell_0 = 15\text{in}$ on his skates moves with velocity $v = 0.8c$ relative to a flat ice surface, approaching a gap of proper width $d_0 = 10\text{in}$.

Skater's perspective (frame S'):

The gap in the ice is Lorentz contracted to a width $d = d_0\sqrt{1-(0.8)^2} = 6\text{in}$, which is shorter than the length $\ell_0 = 15\text{in}$ of his blades. Therefore, the front end A' of the blade will gain support on the far side C of the gap before the back end B' loses support on the near side D. The skater concludes that he will make it across the gap without accident.

Spectator's perspective (frame S):

The blades are Lorentz contracted to length $\ell = \ell_0\sqrt{1-(0.8)^2} = 9\text{in}$, which is shorter than the length $d_0 = 10\text{in}$ of the gap in the ice. Therefore, the back end B' of the blade loses support on the near side D of the gap before the front end A' is able to gain support on the far side C. The spectator concludes that the skater will not make it across the gap without accident.



Analysis:

Event 1: Back end B' of blade enters gap at D.
Event 2: Front end A' of blade exits gap at C.

$$\text{Frame } S': \quad \Delta x' \doteq x'_2 - x'_1 = 15\text{in}, \quad \Delta t' = -(15\text{in} - 6\text{in})/v = -9\text{in}/v.$$
$$\text{Frame } S: \quad \Delta x \doteq x_2 - x_1 = 10\text{in}, \quad \Delta t = (10\text{in} - 9\text{in})/v = 1\text{in}/v.$$

The result $\Delta t' > 0$ (skater's view) suggests safe passage.
The result $\Delta t < 0$ (spectator's view) suggest that an accident will happen.

The answer does not come from quantities that are relative (frame-dependent), but from a quantitiy that is absolute (frame-independent).

Check Lorentz invariant $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$.

$$\text{Frame } S' : \ (\Delta s)^2 = (-9\text{in}/0.8)^2 - (15\text{in})^2 = -(98.4375\text{in})^2 < 0.$$
$$\text{Frame } S : \ (\Delta s)^2 = (1\text{in}/0.8)^2 - (10\text{in})^2 = -(98.4375\text{in})^2 < 0.$$

Conclusion:

Events 1 and 2 have a space-like relationship. They have no definite time-ordering as demonstrated. Such events cannot be causally related.

The skater implies a causal relation between the two events: front end A' finds contact with ice before back end B' loses contact. This implication is fallacious at high speed. The skater assumes the blade is a rigid body, which transports information between ends instantaneously.

The time it takes a light signal to travel from A' to B' is $t_l = 15\text{in/c}$, which longer than the time, $|\Delta t'| = 9\text{in}/v = 11.25\text{in}/c$, it takes the information "we are safe" to travel across the skate

At lower speed, the two events have time-like relationship: event 2 happens before event 1. The skater is seen to be safe from any frame.

The skater is in peril when his speed is high enough to render $(\Delta s)^2$ negative, which makes the two events 1 and 2 space-like.