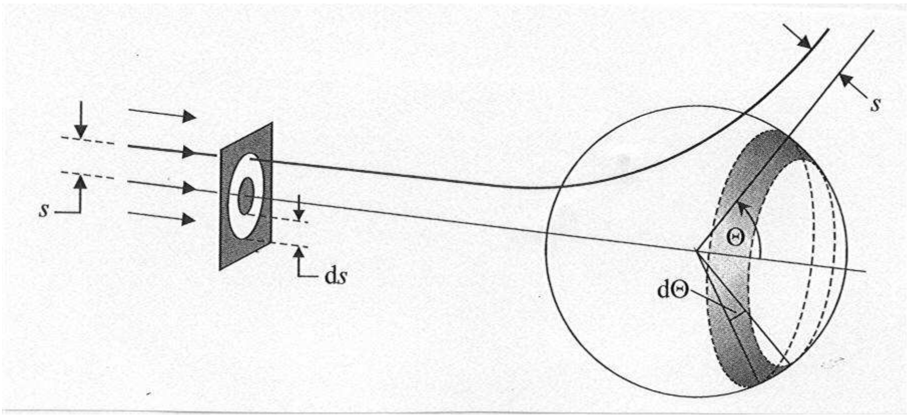
Scattering from central force potential [msl2]



[Goldstein 1981]

Uniform incident beam:

- particles with mass m, energy E;
- intensity I (number of particles crossing unit area in unit time);
- angular momentum depends on impact parameter: $\ell = mv_0 s = \sqrt{2mE}s$.

Scattering cross section:

$$\sigma d\Omega = \frac{\text{number of particles scattered into } d\Omega \text{ per unit time}}{\text{incident beam intensity}}.$$

Element of solid angle: $d\Omega = \sin\theta\, d\theta\, d\phi$.

Total solid angle: $\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi = 4\pi$.

Use of symmetry: $d\Omega = 2\pi \sin\theta\, d\theta$.

Parameters for given $m,\ V(r)$: $\vartheta_0 = 0,\ r_0 = \infty,\ E = \frac{1}{2}mv_0^2,\ \ell = \sqrt{2mE}s$.

Orbital integral: $\vartheta(r) = \ldots$ (see [mln18], [mln20])

Scattering angle: $\theta = \pi - \vartheta(r=\infty) = \theta(E,s) \;\Rightarrow\; s = s(\theta, E)$.

The number of particles scattered into the solid angle $d\Omega$ between θ and $\theta + d\theta$ is equal to the number of incident particles with impact parameter between s and $s + ds$:

$$2\pi I s|ds| = I\sigma d\Omega = 2\pi I \sigma(\theta) \sin\theta\, d\theta.$$

Scattering cross section: $\sigma(\theta) = \dfrac{s}{\sin\theta}\left|\dfrac{ds}{d\theta}\right|$.