Extremum Principles [msl20]

- **Hero of Alexandria** (2^{nd} century BC) : A ray of light traveling from one point to another by reflection from a plane mirror always takes the shortest possible path. $\Rightarrow$ Law of reflection.
- **Fermat** (1657): A ray of light traveling through the interface of optically different media chooses the path that requires the least time. $\Rightarrow$ Law of refraction.
- **Newton, Leibniz, Bernoulli, Euler**: Development of the calculus of variation. Solution of important extremum problems.
- **Maupertuis** (1747): The motion of a dynamical system subject to constraints proceeds in a way that minimizes the action (principle of least action). $\Rightarrow$ Equations for trajectories.
- **Hamilton** (1834): Of all possible paths along which a dynamical system may move between two points within a specified time interval and consistent with any constraints, the actual path followed is that for which the action integral is an extremum. $\Rightarrow$ Equations of motion.

Action integral: $J = \int_{t_1}^{t_2} dt\, L(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)$.

Hamilton's principle: $\delta J = 0$.

Lagrange equations: $\dfrac{\partial L}{\partial q_j} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_j} = 0, \quad j = 1, \ldots, n.$

Lagrange equations are Euler equations for Hamilton's extremum principle.