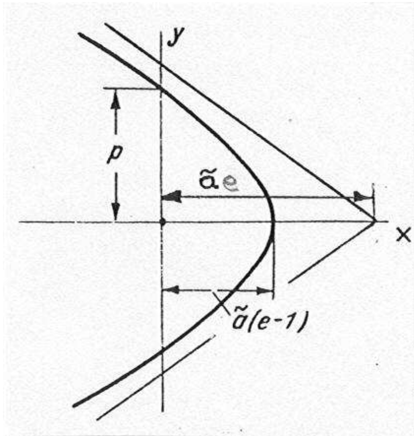
Orbits of the Kepler Problem [msl23]

Orbital equation: $\frac{p}{r} = 1 + e\cos(\vartheta - \vartheta_0)$; $\quad p = \frac{\ell^2}{m\kappa}$, $e = \sqrt{1 + \frac{2E\ell^2}{m\kappa^2}}$, $\kappa = GMm$

(i) **Hyperbolic orbit** ($E > 0,\ e > 1$):

$$\tilde{a} = \frac{p}{e^2 - 1} = \frac{\kappa}{2E}$$

$$r_{min} = \frac{p}{e+1} = \tilde{a}(e-1) \quad \text{(perihelion)}$$



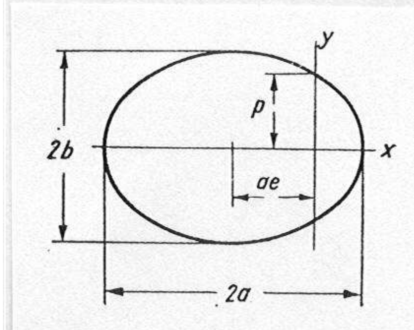
(ii) **Elliptic orbit** ($E < 0,\ e < 1$):

$$a = \frac{p}{1 - e^2} = \frac{\kappa}{2|E|} \quad \text{(semi-major axis)}$$

$$b = \frac{p}{\sqrt{1 - e^2}} = \frac{\ell}{\sqrt{2m|E|}} \quad \text{(semi-minor axis)}$$

$$r_{min} = \frac{p}{1+e} = a(1-e) \quad \text{(perihelion)}$$

$$r_{max} = \frac{p}{1-e} = a(1+e) \quad \text{(aphelion)}$$



(iii) **Parabolic orbit** ($E = 0,\ e = 1$):

For $E \to 0^+$ at $\ell \neq 0$: $\tilde{a} \to \infty$ (asymptotes become parallel)

For $E \to 0^-$ at $\ell \neq 0$: $a \to \infty,\ b \to \infty,\ r_{max} \to \infty,\ r_{min} \to \frac{p}{2} = \frac{\ell^2}{2m\kappa}$

(iv) **Circular orbit** ($E = -m\kappa^2/2\ell^2,\ e = 0$):

$$a = b = p = r_{min} = r_{max} = \frac{\kappa}{2|E|} = \frac{\ell^2}{m\kappa}$$