Eulerian Angles of Rotation [msl25]

The coordinate system (x_1, x_2, x_3) has its axes fixed to the rigid body.
The coordinate system (x'_1, x'_2, x'_3) has the same origin but its axes kept parallel to those of an inertial system.

The orthogonal transformation between the two coordinate systems is constructed from three successive rotations by Eulerian angles ϕ, θ, ψ about three different axes.

$$\mathbf{O} = \mathbf{O}_\psi \cdot \mathbf{O}_\theta \cdot \mathbf{O}_\phi.$$

$$\mathbf{O}_\phi = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{rotation about } x'_3\text{-axis.}$$

$$\mathbf{O}_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \qquad \text{rotation about line of nodes.}$$

$$\mathbf{O}_\psi = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{rotation about } x_3\text{-axis.}$$

$$\mathbf{O} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix}.$$