

## Phase Portraits of Conservative Systems [ms15]

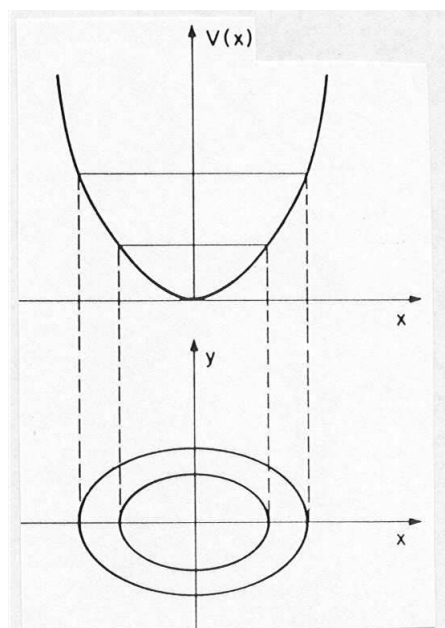
### Harmonic oscillator

$$\dot{x} = y, \quad \dot{y} = -\frac{k}{m}x.$$

$$E(x, y) = \frac{1}{2}my^2 + \frac{1}{2}kx^2 = \text{const.}$$

$$\Rightarrow \frac{x^2}{2E/k} + \frac{y^2}{2E/m} = 1 \quad (\text{ellipses}).$$

Elliptic fixed point at  $(x, y) = (0, 0)$ .



### Point mass in double-well potential

2 elliptic fixed points (potential minima).

Stable equilibrium positions.

1 hyperbolic fixed point (potential maximum).

Unstable equilibrium position.

1 separatrix.

[Arnold 1978]

