[mex155] Kinetic energy in Lagrangian mechanics

Show that the kinetic energy of a dynamical system of N particles subject to k scleronomic constraints $\mathbf{r}_i = \mathbf{r}_i(q_1, \ldots, q_n),\ i = 1, \ldots, N$ is a homogeneous quadratic function of the generalized coordinates:

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i |\dot{\mathbf{r}}_i|^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} a_{jk} \dot{q}_j \dot{q}_k.$$

Identify the coefficients a_{jk} and show that the following relation holds:

$$\sum_l \dot{q}_l \frac{\partial T}{\partial \dot{q}_l} = 2T.$$

Solution: