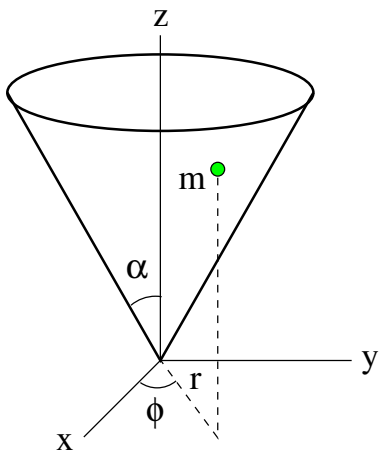
[mex159] Particle sliding inside cone: normal force of constraint

Consider a conical surface with vertical axis (z) and apex with angle 2α at the bottom in a uniform gravitational field g. A particle of mass m is free to slide on the inside of the cone.
(a) Write the equation of holonomic constraint $f(z, r, \phi) = 0$ between the cylindrical coordinates and the Lagrangian $L(z, r, \phi)$
(b) Derive the three Lagrange equations. Along with the equation $f = 0$, they determine the generalized coordinates z, r, ϕ and the Lagrange multiplier λ.
(c) By using the conservation law $p_\phi = mr^2\dot{\phi} = \text{const}$, solve the equations of motion (without integrating $\ddot{z}$ and $\ddot{r}$) for the Lagrange multiplier $\lambda(p_\phi, r)$ and infer from it the three components $Q_z = \lambda\partial f/\partial z$, $Q_r = \lambda\partial f/\partial r$, $Q_\phi = \lambda\partial f/\partial\phi$ of the normal force of constraint.



Solution: