[mex16] Projectile in resistive medium

A particle of mass m is projected vertically upward with initial velocity v_0 against a uniform gravitational field g and against a resistive force
(a) $F(v) = -\beta v$, (b) $F(v) = -\gamma v^2$.
(i) In each case, calculate the following results for the maximum height h reached by the particle and the time T it takes to get there:

$$h_\beta = \frac{m}{\beta}\left[v_0 - \frac{mg}{\beta}\ln\left(1+\frac{\beta}{mg}v_0\right)\right], \quad T_\beta = \frac{m}{\beta}\ln\left(1+\frac{\beta}{mg}v_0\right).$$

$$h_\gamma = \frac{m}{2\gamma}\ln\left(1+\frac{\gamma}{mg}v_0^2\right), \quad T_\gamma = \sqrt{\frac{m}{g\gamma}}\arctan\left(\sqrt{\frac{\gamma}{mg}}v_0\right).$$

(ii) Show that these results coincide in the limits $\beta \to 0$ and $\gamma \to 0$, respectively.

Solution: