[mex167] Mechanical refraction

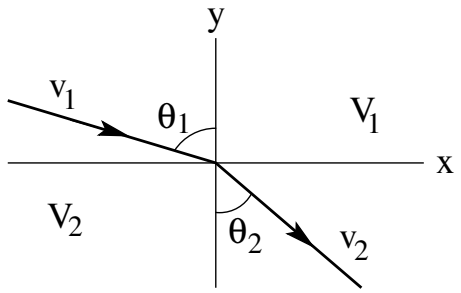
Snell's law for the refraction of a ray of light states that the ratio of the sines of the angles of incidence and refraction is equal to the ratio of phase velocities in the two media or, equivalently, equal to the ratio of the indices of refraction in the opposite media:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

A particle of mass m moving in the xy-plane is subject to a potential energy which assumes the constant value V_1 at $y \geq 0$ and the constant value V_2 at $y < 0$. Let us assume that $V_2 < V_1$. Use conversation laws to show that if the particle approaches the x-axis with speed v_1 at an angle θ_1 as shown, it will proceed with a different speed v_2 at a different angle θ_2 after crossing the line where the potential energy changes abruptly. Show in particular that the relation,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1}, \quad v_2 = v_1\sqrt{1 + \frac{V_1 - V_2}{K_1}}, \quad K_1 = \frac{1}{2}mv_1^2,$$

between the two angles holds, implying an inverse relationship between velocities and angles.



Solution: