[mex221] Momentum conservation

Two particles with equal masses m as measured when at rest are undergoing an inelastic collision as shown in the lab frame S. Conservation of total momentum implies

$$m(v)\mathbf{v} + m(0)0 = M(\bar{v})\bar{\mathbf{v}},$$

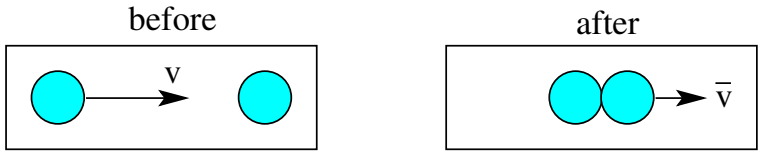
where $v = 2\bar{v}/(1 + \bar{v}^2/c^2)$ from [mln63]. Use the requirement that the total momentum is also conserved in frame S'' that moves with relative velocity $\mathbf{u}$ perpendicular to $\mathbf{v}$ to infer the relation,

$$m(v'')\mathbf{u} + m(u)\mathbf{u} = M(\bar{v}'')\mathbf{u},$$

where $v'' = \sqrt{v^2 + u^2(1 - v^2/c^2)}$ and $\bar{v}'' = \sqrt{\bar{v}^2 + u^2(1 - \bar{v}^2/c^2)}$. In the limit $u \to 0$, this becomes the relativistic relation,

$$m(v) + m(0) = M(\bar{v}),$$

between the individual masses before the collision and composite mass after the collision.



Solution: