[mex238] Decay of particle: directions in lab frame I

A particle of mass M and velocity $\mathbf{v}_0$ (in the lab frame) decays into two particles of masses m_1, m_2 by releasing a total decay energy ϵ. Assume that mass-energy conversion is negligible ($M = m_1 + m_2$) and that the resulting momenta of the decay products are nonrelativistic. Show that the angles of the directions of the decay products relative to the forward direction of the original particle satisfy the following relation:

$$\frac{2\epsilon \sin^2(\bar{\theta}_1 + \bar{\theta}_2)}{(m_1 + m_2)v_0^2} = \frac{m_1}{m_2}\sin^2\bar{\theta}_1 + \frac{m_2}{m_1}\sin^2\bar{\theta}_2 - 2\sin\bar{\theta}_1 \sin\bar{\theta}_2 \cos(\bar{\theta}_1 + \bar{\theta}_2).$$

Explain what this relation implies in the two limits $v_0 \to 0$ and $v_0 \to \infty$.

Solution: