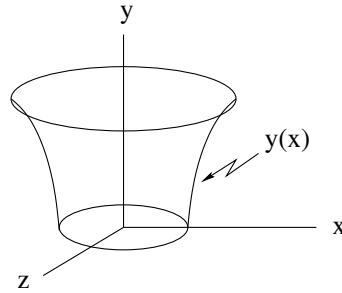


[mex27] Economy plastic cup

A manufacturer of plastic cups receives an order for cups of given height  $h$ , radius  $r_T$  at the top, radius  $r_B$  at the bottom, and uniform material thickness. Determine the profile  $y(x)$  at  $z = 0$  of the cup which minimizes the amount of plastic needed for each cup. The specifications are  $r_T = 3.5\text{cm}$ ,  $r_B = 3.0\text{cm}$ ,  $h = 1.5\text{cm}$ .

- Find the functional  $f[y, y'; x]$  which enters the integral expression for the curved area  $A$ .
- Infer the profile  $y(x)$  as the solution (with two integration constants) of Euler's equation .
- Determine the integration constants to match the manufacturer's specifications.
- Evaluate the (curved) area  $A$  and the volume capacity  $V$  of the cup. Use units  $\text{cm}^2$  and  $\text{cm}^3$ .



**Solution:**