

[mex274] Kepler orbital equation

In [mln18] we have derived the orbital integral for a generic power-law central-force potential, $V(r) = -\kappa/r^\alpha$, in the form

$$\vartheta - \vartheta_0 = - \int_{u_0}^u \frac{du}{\sqrt{\frac{2mE}{\ell^2} + \frac{2m\kappa}{\ell^2} u^\alpha - u^2}}, \quad u \doteq \frac{1}{r}.$$

Use the parametrization,

$$p \doteq \frac{\ell^2}{m\kappa}, \quad e \doteq \sqrt{1 + \frac{2E\ell^2}{m\kappa^2}}, \quad \kappa \doteq GMm,$$

for the case of the gravitational potential, $V(r) = -GMm/r$, and show that the integration yields the orbital equation,

$$\frac{p}{r} = 1 + e \cos(\vartheta - \vartheta_0),$$

which is further analyzed in [msl23].

Solution: