

### [mex279] Soap film between parallel circles

When a film of soap is formed that connects two parallel circular wires positioned and oriented as shown, the surface tension makes it settle in a shape that minimizes its area.

(a) Express the area  $A$  of the film as an integral of the functional  $f[r, r'; x]$ .

(b) Show that Euler's equation for that functional can be reduced to the following ODE of the radial profile  $r(x)$ :

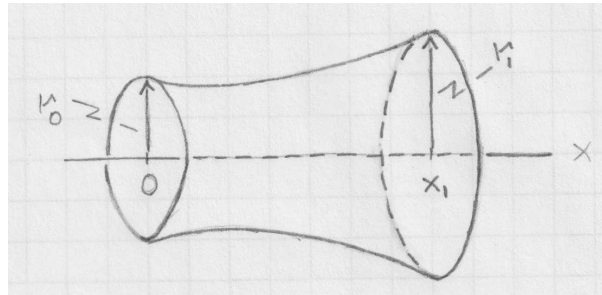
$$rr'' - (r')^2 = 1. \quad (1)$$

(c) Show that (1) has a two-parameter solution of the form,

$$r(x) = a \cosh\left(\frac{x}{a} + c\right). \quad (2)$$

(d) Determine the parameters for a film with specifications:  $r_0 = 1.0\text{cm}$ ,  $r_1 = 1.2\text{cm}$ ,  $x_1 = 1.4\text{cm}$ .

(e) Calculate the area  $A$  of the film for these specifications.



**Solution:**