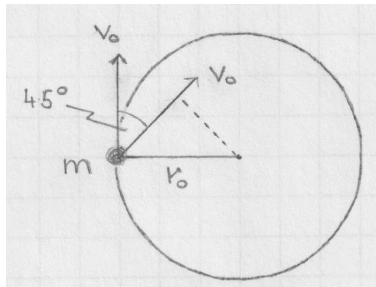


### [mex281] Sudden turn

Consider a particle of mass  $m$  in a stable circular orbit at radius  $r_0$  of a central force potential  $V(r)$ . An external agent causes the particle to make a sudden  $45^\circ$ -turn with no change of speed  $v_0$  toward the center of the potential in the plane of the orbit. Describe the nature of the orbit after the turn by answering questions (a)-(d) separately for the two specific power-law potentials,

$$(i) V(r) = \frac{1}{2}kr^2, \quad (ii) V(r) = -\frac{\kappa}{r}.$$

- Express the kinetic energy  $T_0$ , the potential energy  $V_0$ , the total energy  $E_0$ , and the angular momentum  $\ell_0$  before the intervention as functions of  $r_0$ , noting that  $v_0$  is a function of  $r_0$ .
- Express the kinetic energy  $T_1$ , the potential energy  $V_1$ , the total energy  $E_1$ , and the angular momentum  $\ell_1$  right after the intervention as functions of  $r_0$ .
- Determine the apsidal radii  $r_{\min}$  and  $r_{\max}$  of the elliptic orbit as functions of  $r_0$ .
- Express the semimajor axis  $a$  and the semiminor axis  $b$  of the ellipse as functions of the apsidal radii  $r_{\min}$  and  $r_{\max}$ .



**Solution:**