

[mex31] Brachistochrone problem II

A particle of mass m slides from rest at the origin of the coordinate system down to the point E along a frictionless path.

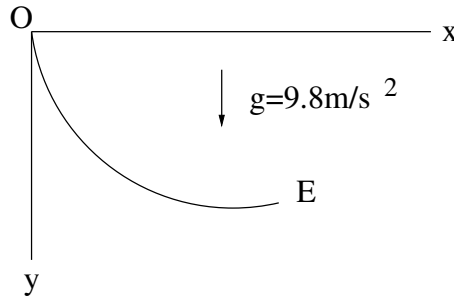
(a) Use the calculus of variation to determine the path along which the particle arrives at E in the shortest time t_E . Construct an integral expression for t_E with a functional $f[x, x'; y]$. Infer from it the differential expression,

$$dx = \frac{y dy}{\sqrt{2ay - y^2}}, \quad (1)$$

for the optimal path. This expression may not produce a single-valued function $x(y)$.

(b) Circumnavigate the problem with the parametric representation $y(\phi) = a(1 - \cos \phi)$. Infer the dependence $x(\phi)$ from (1) and the dependence $t(\phi)$ from the differential of the original integral expression for t_E .

(c) Use the parameters of [mex30], $x_E = 200\text{m}$, $y_E = 100\text{m}$, to first determine the values a and ϕ_E , and then to determine the time t_E it takes the particle to slide along the optimal path. It should be shorter than the time found in [mex30] for the optimal path consisting of two straight segments.



Solution: