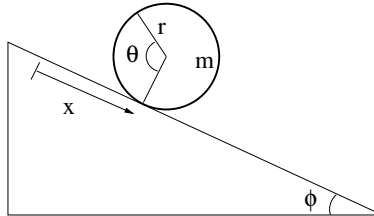


**[mex32] Static frictional force of constraint**

Consider a hoop of mass  $m$  and radius  $r$  rolling without slipping down an incline.

(a) Determine the Lagrangian  $L(x, \dot{x})$  of this one-degree-of-freedom system. Derive from it the Lagrange equation and its solution for initial condition  $x_0 = 0, \dot{x}_0 = 0$ .

(b) Determine the alternative Lagrangian  $L(x, \theta, \dot{x}, \dot{\theta})$  and the holonomic constraint  $f(x, \theta) = 0$  that must accompany it. Derive the associated three equations of motion for the two unknown dynamical variables  $x, \theta$  and the undetermined Lagrange multiplier  $\lambda$ . Solve these equations for the same initial conditions as in (a) and determine the static frictional force of constraint between the hoop and the incline.



**Solution:**