[mex46] Orbit of the inverse-square potential at large angular momentum

Consider the central force potential $V(r) = -\kappa/r^2$. If $\kappa < \ell^2/2m$, all orbits are unbounded and have energies $E > 0$. (a) Show that the orbits can be expressed in the form

$$\frac{1}{r} = \sqrt{\frac{2mE}{\ell^2 - 2m\kappa}} \cos\left(\vartheta\sqrt{1 - \frac{2m\kappa}{\ell^2}}\right).$$

(b) Determine the total angle an orbit describes between the incoming and outgoing asymptotes.

Solution: