[mex5] Periodic motion in power-law potentials

Use the expression,

$$\tau = 2\int_{x_{min}}^{x_{max}} \frac{dx}{\sqrt{2[E - V(x)]/m}},$$

readily inferred from [mln4], to calculate the dependence on the amplitude x_{max} of the period τ for the motion of a particle with mass m moving in power-law potentials of the following kind:
(a) quadratic potential $V_2(x) = \frac{1}{2}kx^2$,
(b) quartic potential $V_4(x) = \frac{1}{4}\alpha x^4$,
(c) linear potential $V_1(x) = \beta|x|$.
Note that in one case the period is independent of the amplitude, which is the hall mark of harmonic oscillations.

Solution: