[mex54] Circular orbits of the Yukawa potential

A particle of mass m moves in the Yukawa potential $V(r) = -(k/r)e^{-r/\rho}$, where k is a measure for the strength and ρ a measure for the range of the interaction. Circular orbits exist only if the angular momentum ℓ does not exceed a certain value ℓ_{max}. For any value $\ell < \ell_{max}$, there exist two circular orbits, one stable orbit at radius $R_S(\ell)$ and one unstable orbit at radius $R_U(\ell)$.
(a) Establish the (dimensionless) relation, $\ell/\sqrt{mk\rho} = f(R/\rho)$ which determines the values R_U and R_S for given $\ell < \ell_{max}$ and produce a sketch of that function.
(b) Determine the value of the (conveniently scaled) maximum angular momentum, $\ell_{max}/\sqrt{mk\rho}$.
(c) Determine the value $R_S(\ell_{max})/\rho = R_U(\ell_{max})/\rho$ of the (merged) scaled radii at maxmum angular momentum.

Solution: