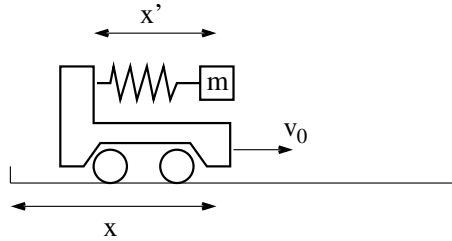


[mex77] **Hamiltonian: conserved quantity or total energy?**

A harmonic oscillator (mass  $m$ , spring constant  $k$ ) is attached to a cart and oscillates with displacement  $x'$  relative to the cart, while the cart moves with constant velocity  $\mathbf{v}_0$  relative to the ground.

- Construct the Lagrangian  $L$  of the oscillator using the coordinate  $x$  and derive the associated Lagrange equation. Construct the Hamiltonian  $H$  from  $L$ .
- Construct the Lagrangian  $L'$  of the oscillator using the coordinate  $x'$  and derive the associated Lagrange equation. Construct the Hamiltonian  $H'$  from  $L'$ .
- Show that the Lagrange equations obtained in (a) and (b) are equivalent.
- Which Hamiltonian, if any, represents the total energy of the oscillator?
- Which Hamiltonian, if any, represents a conserved quantity?



**Solution:**