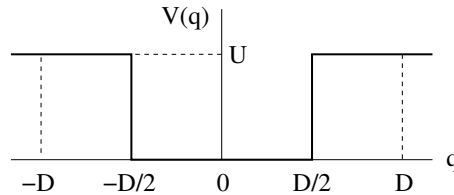


[mex95] Bounded motion in piecewise constant periodic potential

Consider a particle of mass m moving in the potential $V(q) = 0$ for $0 < |q| < D/2$ and $V(q) = U$ for $D/2 < |q| < D$ with periodicity $V(q + 2D) = V(q)$. For energies $E < U$ the motion is bounded. Solve this dynamical problem via transformation $(q, p) \rightarrow (\theta, J)$ to action-angle coordinates for motion with initial conditions $q(0) = 0$, $p(0) > 0$. (a) Find the function $K(J)$, which expresses the Hamiltonian as a function of the action coordinate. (b) Find the period $T(E) \doteq 2\pi/\omega(J)$ of the librational motion. (c) Find the function $q(\theta, J)$ for $0 < \theta < 2\pi$. (d) Sketch in one diagram the functions $J = \text{const}$ and $p(t)$ for $0 < t < T$. (e) Sketch in a second diagram the functions $q(t)$ and $\theta(t)$ for $0 < t < T$. Assume in both diagrams that the particle starts at $q = 0$ moving to the right.



Solution: