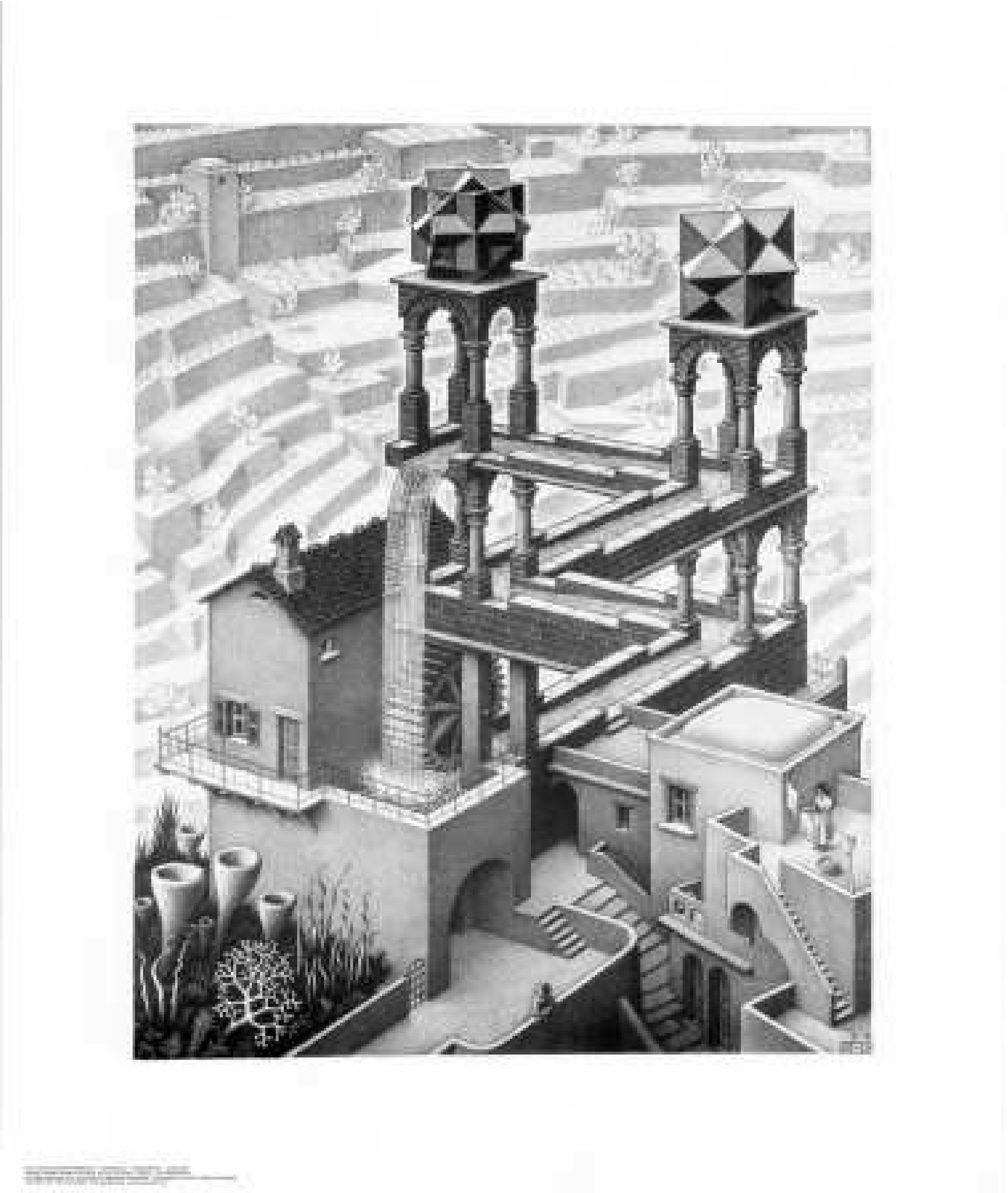
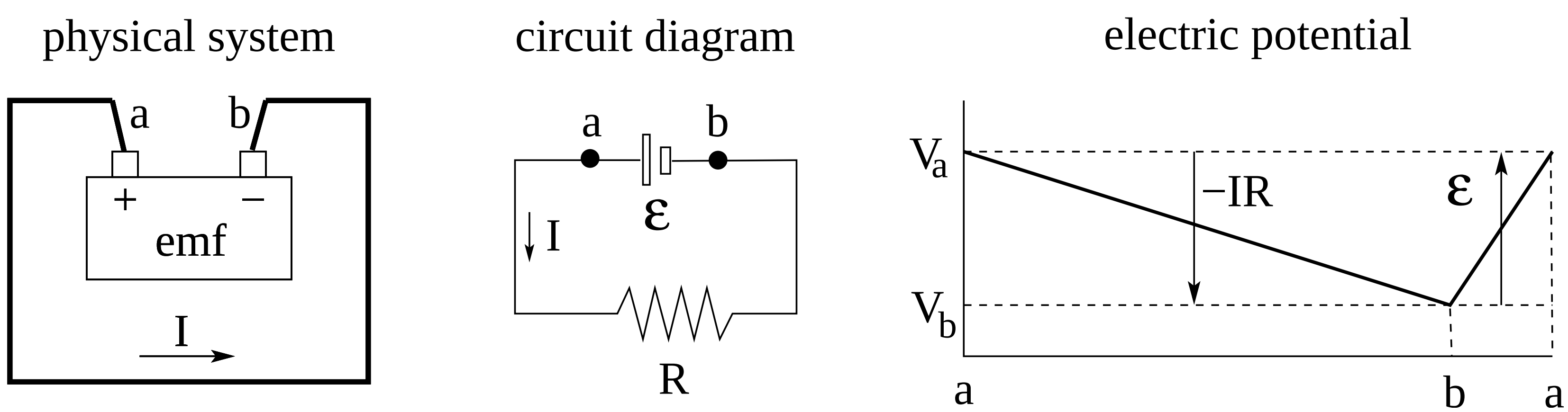
# M. C. Escher: Waterfall



# Direct Current Circuit

Consider a wire with resistance $R = \rho\ell/A$ connected to a battery.

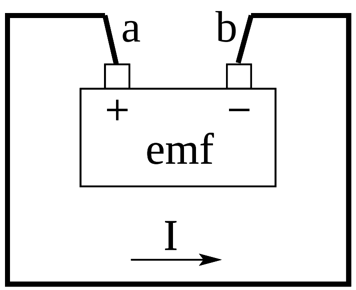
- **Resistor rule**: In the direction of $I$ across a resistor with resistance $R$, the electric potential drops: $\Delta V = -IR$.
- **EMF rule**: From the $(-)$ terminal to the $(+)$ terminal in an ideal source of emf, the potential rises: $\Delta V = \mathcal{E}$.
- **Loop rule**: The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero: $\sum \Delta V_i = 0$.



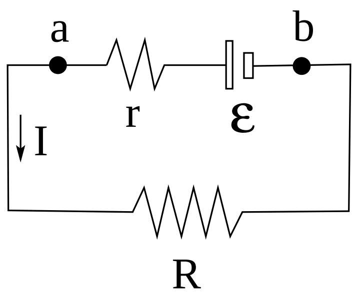
# Battery with Internal Resistance

- Real batteries have an internal resistance $r$.
- The terminal voltage $V_{ba} \equiv V_a - V_b$ is smaller than the emf $\mathcal{E}$ written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \; I = \dfrac{\mathcal{E}}{R + r}$
- Current from terminal voltage: $V_{ba} = \mathcal{E} - Ir = IR \quad \Rightarrow \; I = \dfrac{V_{ba}}{R}$

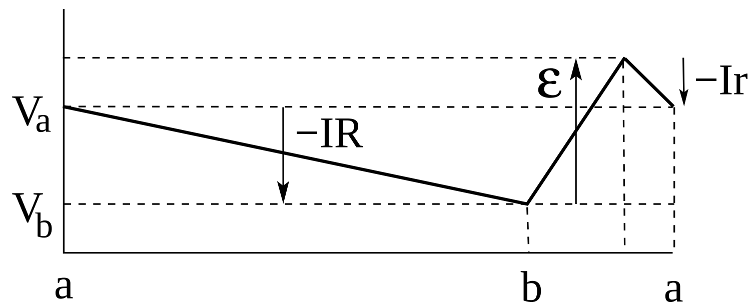
physical system



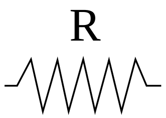
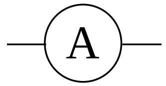
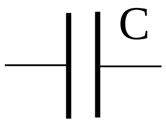
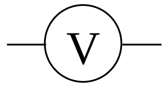
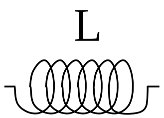
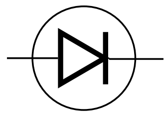
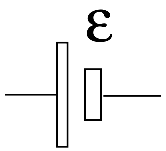
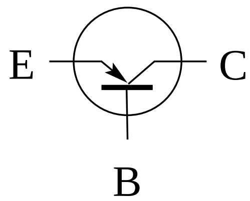

circuit diagram




electric potential



# Symbols Used in Cicuit Diagrams

| | | | |
|---|---|---|---|
|  | resistor |  | ammeter (connect in series) |
|  | capacitor |  | voltmeter (connect in parallel) |
|  | inductor |  | diode |
|  | emf source |  | transistor |

# Resistor Circuit (4)

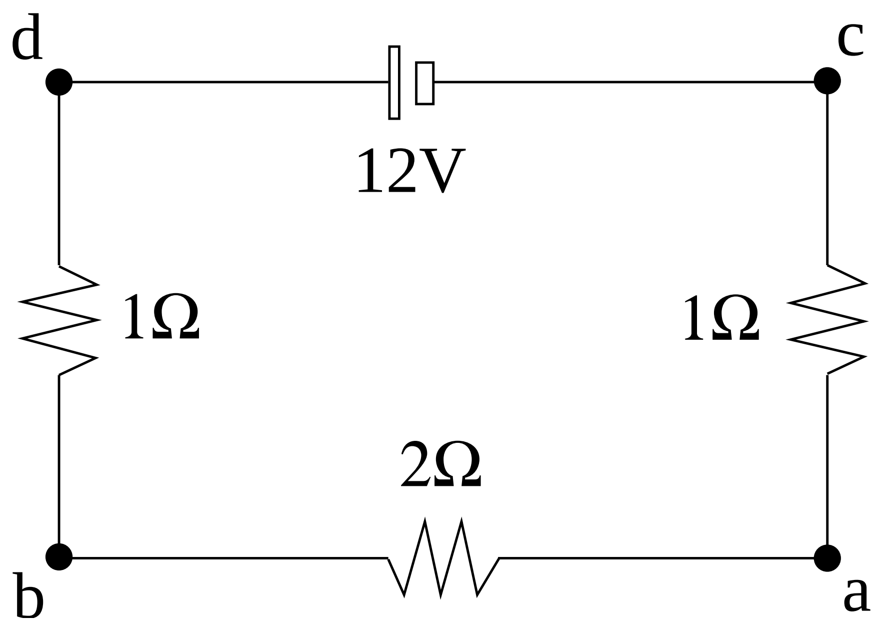
Consider the resistor circuit shown.

(a) Find the direction of the current $I$ (cw/ccw).

(b) Find the magnitude of the current $I$.

(c) Find the voltage $V_{ab} = V_b - V_a$.

(d) Find the voltage $V_{cd} = V_d - V_c$.



# Resistor Circuit (5)

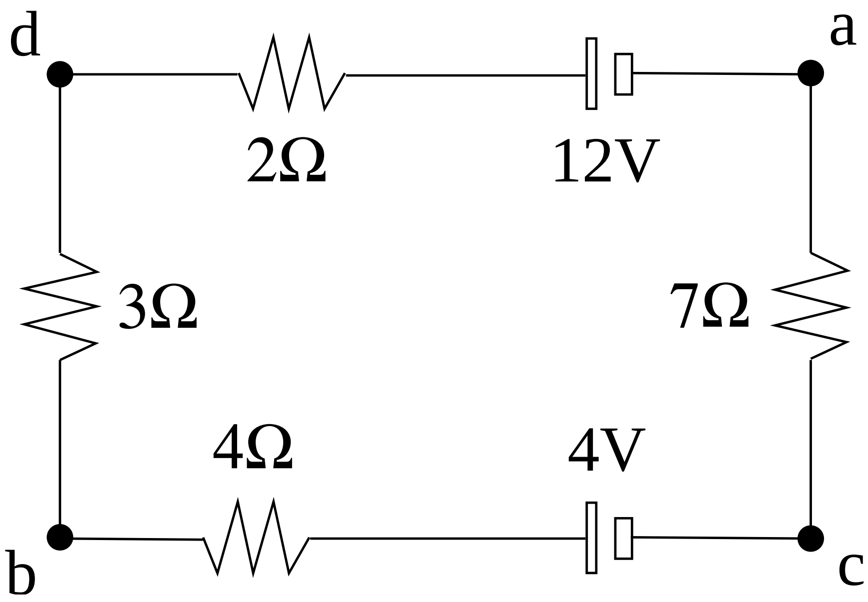
Consider the resistor circuit shown.

(a) Find the direction (cw/ccw) of the current $I$ in the loop.

(b) Find the magnitude of the current $I$ in the loop.

(c) Find the potential difference $V_{ab} = V_b - V_a$.

(d) Find the voltage $V_{cd} = V_d - V_c$.



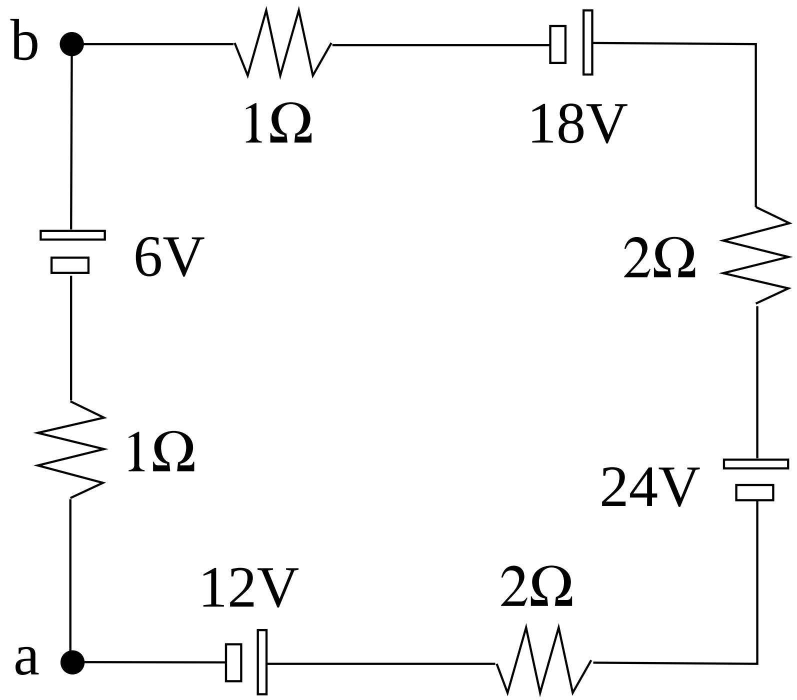
# Resistor Circuit (6)

Consider the resistor circuit shown.

(a) Guess the current direction (cw/ccw).

(b) Use the loop rule to determine the current.

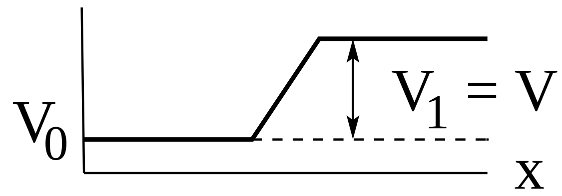
(c) Find $V_{ab} \equiv V_b - V_a$ along two different paths.

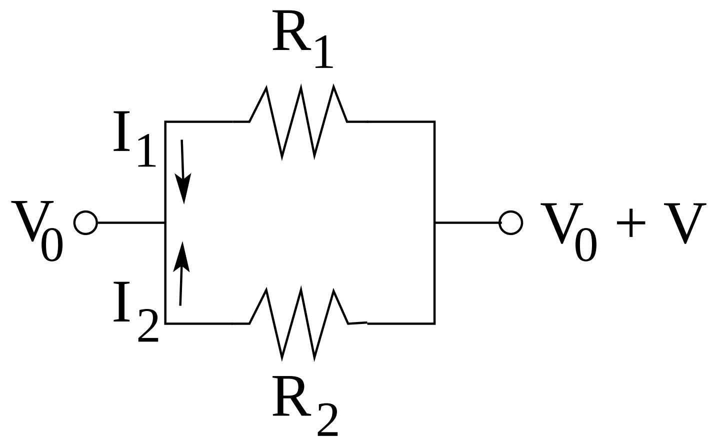


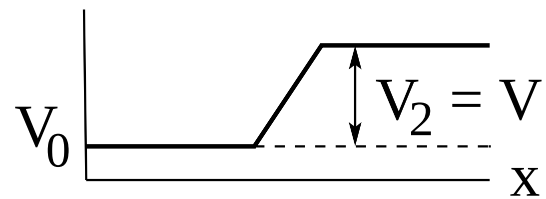
# Resistors Connected in Parallel

Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



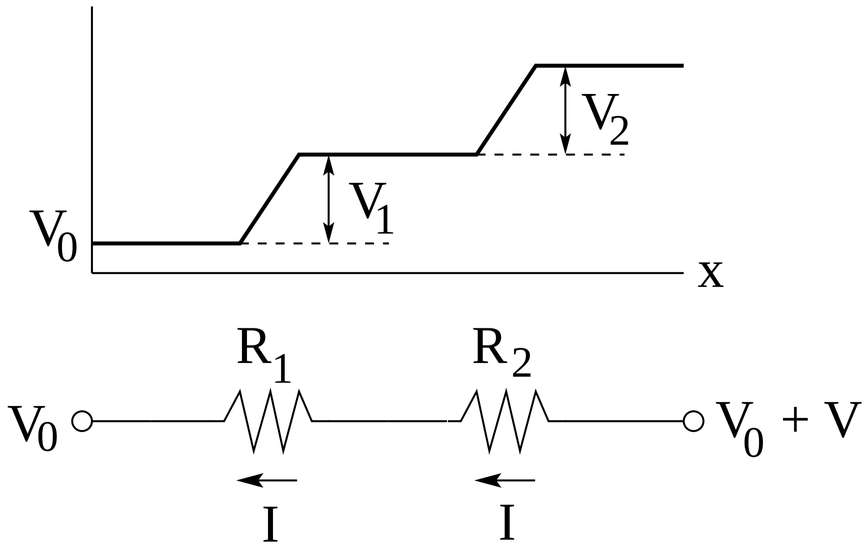



# Resistors Connected in Series

Find the equivalent resistance of two resistors connected in series.

- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow \; R = R_1 + R_2$



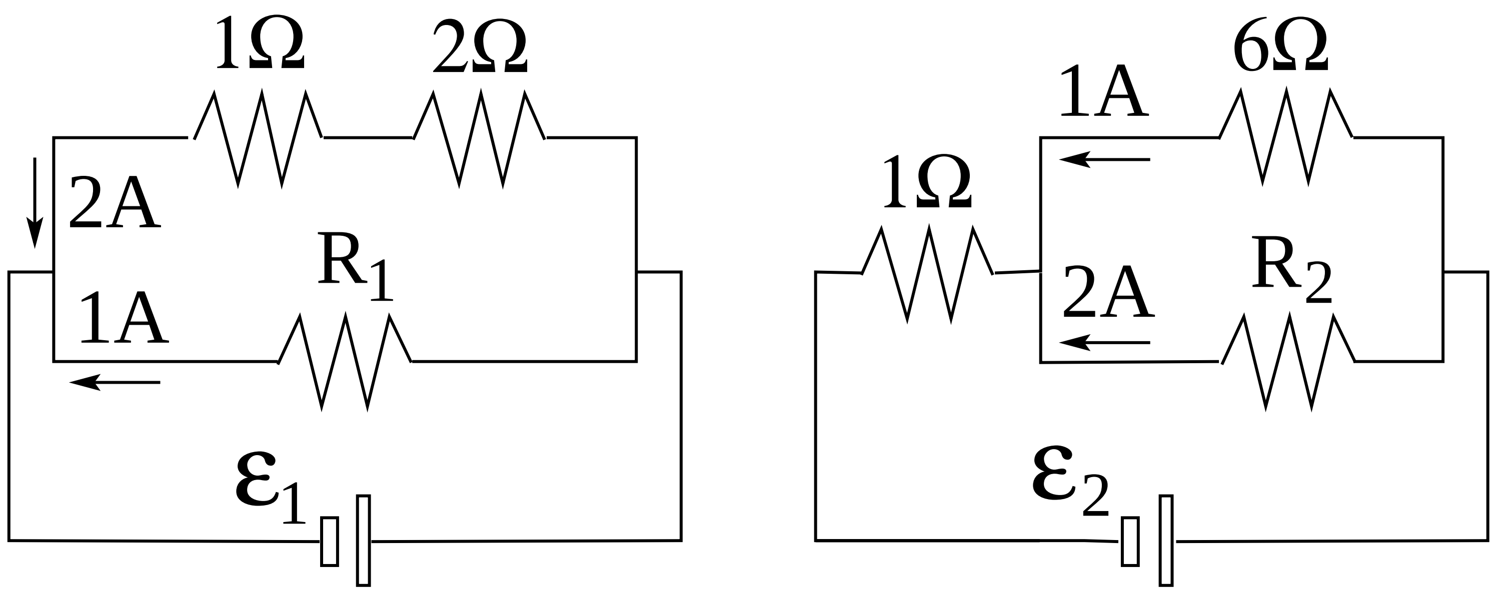
Consider the two resistor circuits shown.

(a) Find the resistance $R_1$.

(b) Find the emf $\mathcal{E}_1$.

(c) Find the resistance $R_2$.

(d) Find the emf $\mathcal{E}_2$.



# Resistor Circuit (2)

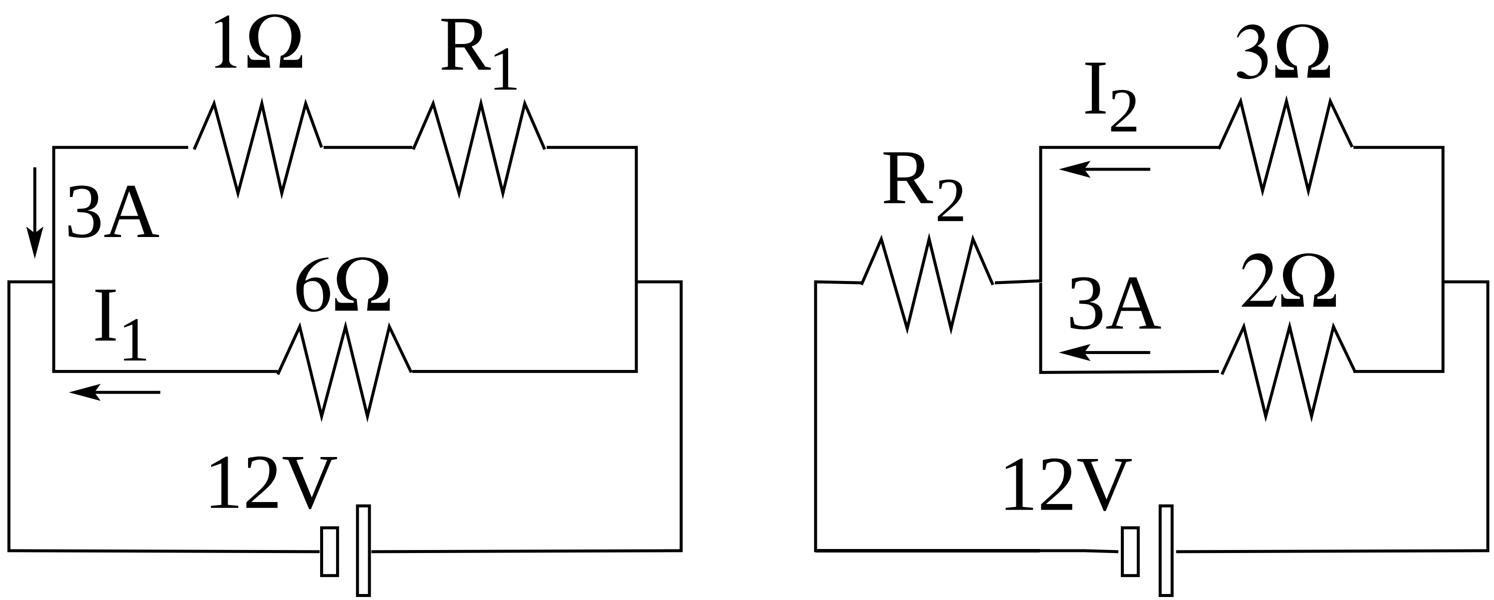
Consider the two resistor circuits shown.

(a) Find the resistance $R_1$.

(b) Find the current $I_1$.

(c) Find the resistance $R_2$.

(d) Find the current $I_2$.



# Resistor Circuit (3)

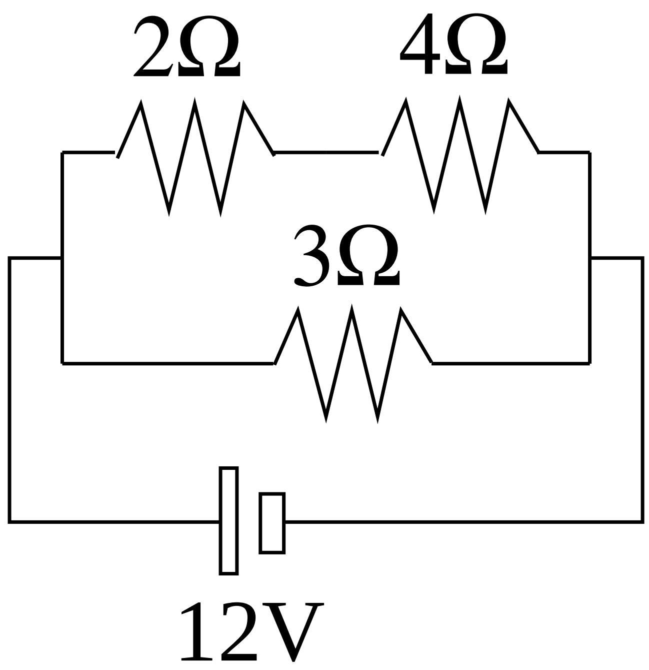
Consider the rsistor and capacitor circuits shown.

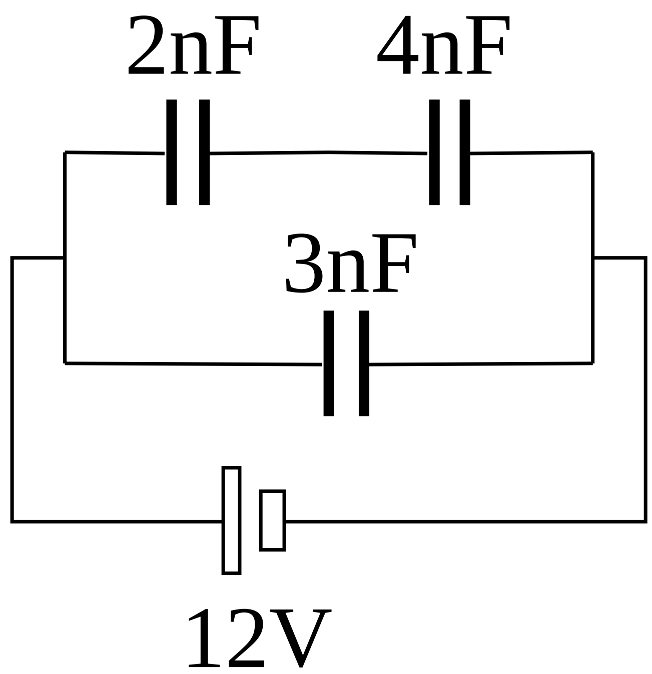
(a) Find the equivalent resistance $R_{eq}$.

(b) Find the power $P_2, P_3, P_4$ dissipated in each resistor.

(c) Find the equivalent capacitance $C_{eq}$.

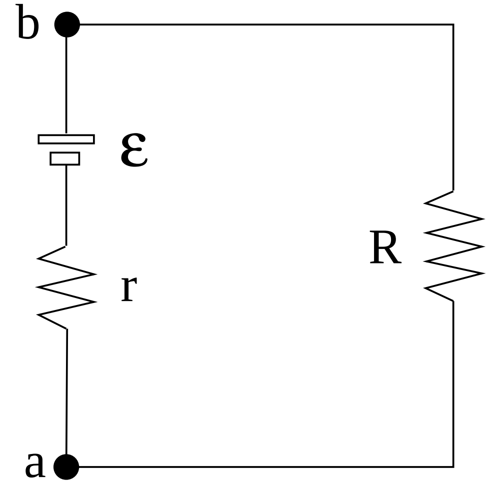
(d) Find the energy $U_2, U_3, U_4$ stored in each capacitor.



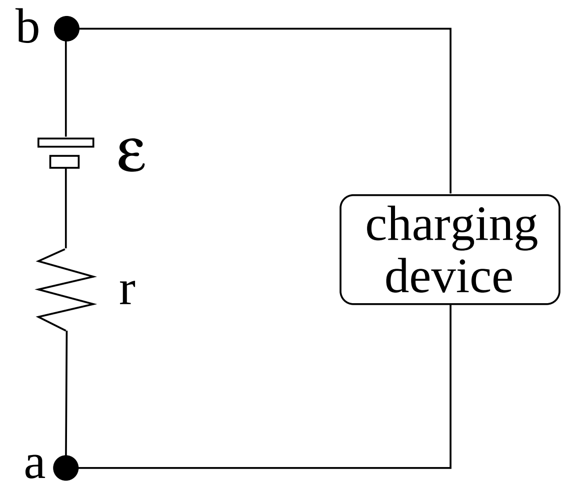
## Battery in use

- Terminal voltage: $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery: $P = V_{ab}I = \mathcal{E}I - I^2r$
  - Power generated in battery: $\mathcal{E}I$
  - Power dissipated in battery: $I^2r$
- Power dissipated in resistor: $P = I^2R$




## Battery being charged:

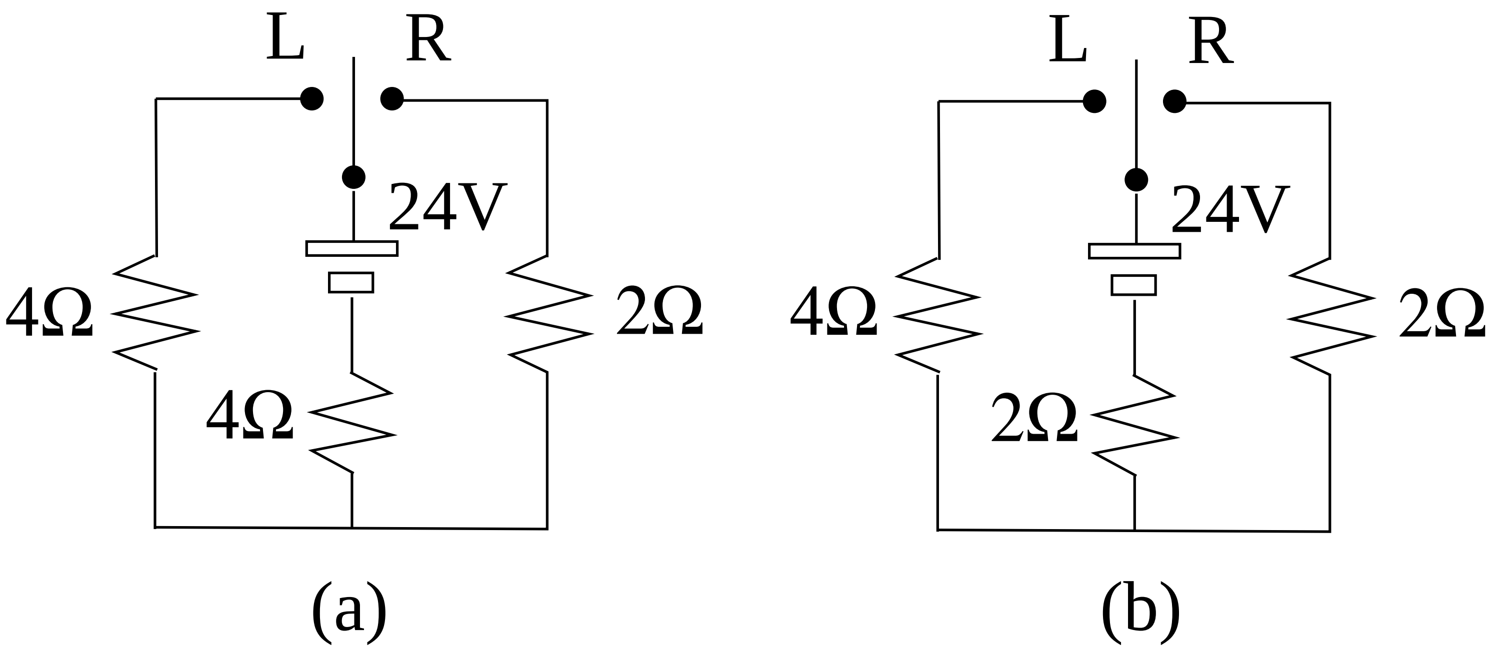
- Terminal voltage: $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device: $P = V_{ab}I$
- Power input into battery: $P = \mathcal{E}I + I^2r$
  - Power stored in battery: $\mathcal{E}I$
  - Power dissipated in battery: $I^2r$



# Resistor Circuit (7)

Consider two 24V batteries with internal resistances (a) $r = 4\Omega$, (b) $r = 2\Omega$.

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?

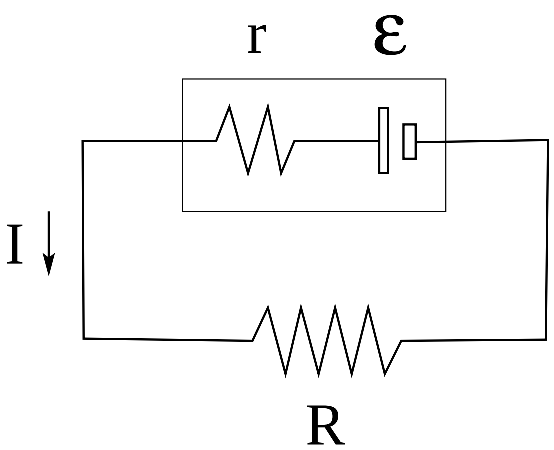


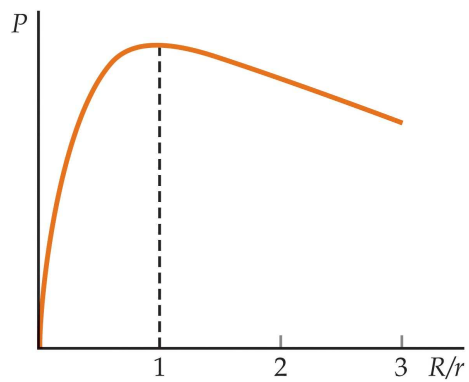
# Impedance Matching

A battery providing an emf $\mathcal{E}$ with internal resistance $r$ is connected to an external resistor of resistance $R$ as shown.

For what value of $R$ does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \; I = \dfrac{\mathcal{E}}{R+r}$
- Power delivered to external resistor: $P = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$
- Condition for maximum power: $\dfrac{dP}{dR} = 0 \quad \Rightarrow \; R = r$

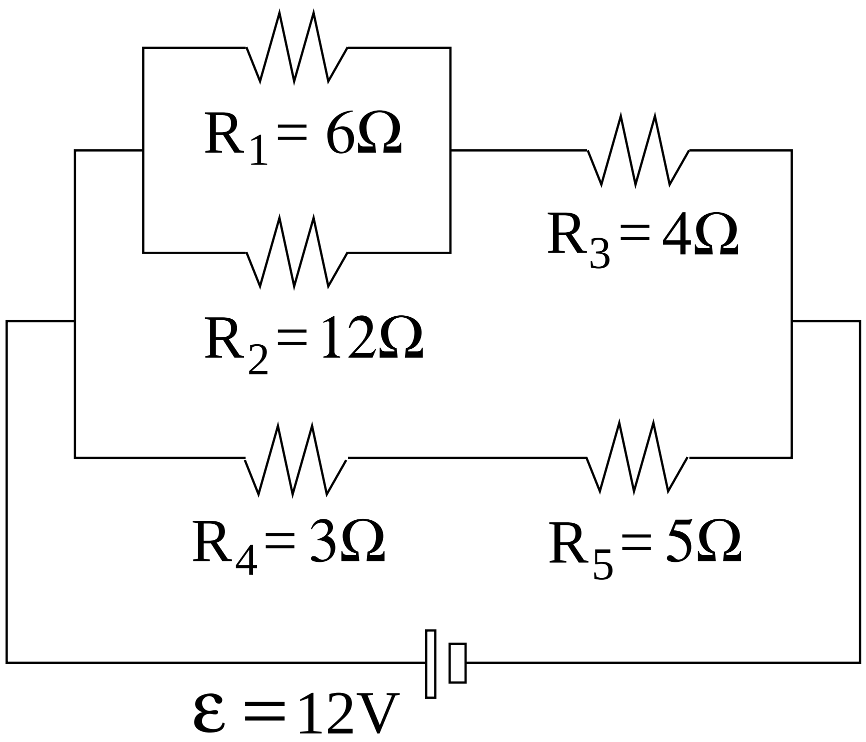


# Resistor Circuit (8)

Consider the circuit of resistors shown.

- Find the equivalent resistance $R_{eq}$.
- Find the currents $I_1, \ldots, I_5$ through each resistor and the voltages $V_1, \ldots, V_5$ across each resistor.
- Find the total power $P$ dissipated in the circuit.



## Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

## Junction Rule

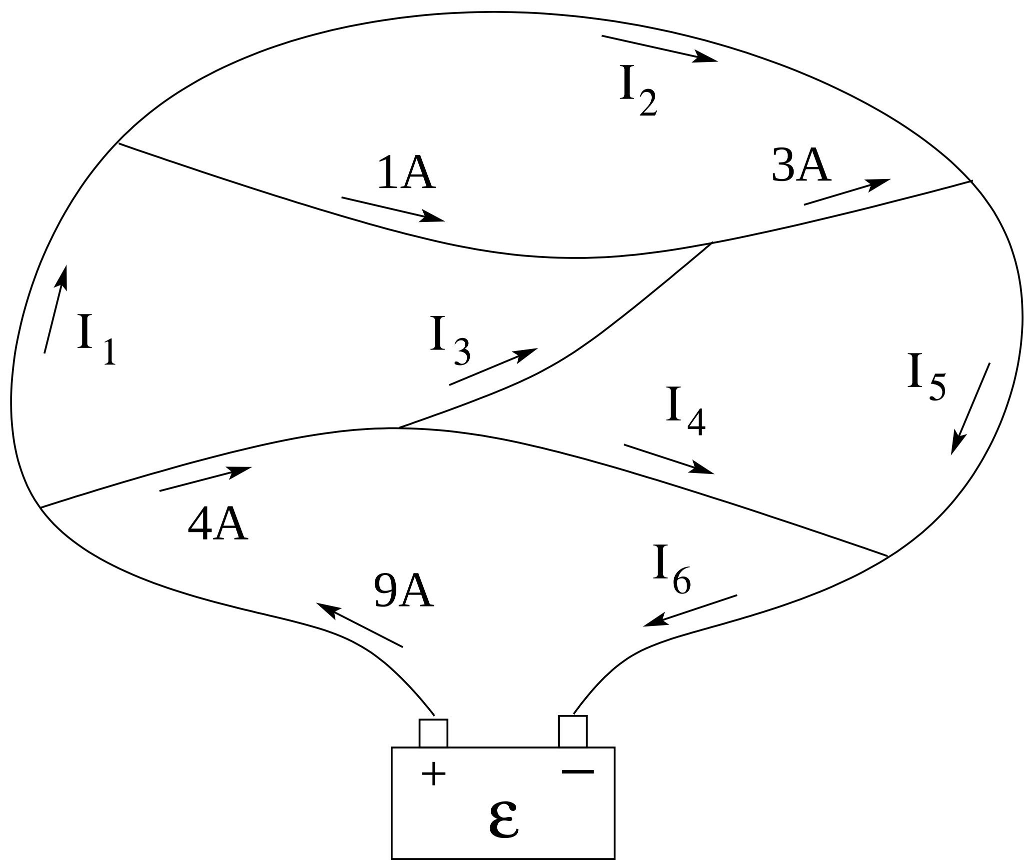
- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

## Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

# Applying the Junction Rule

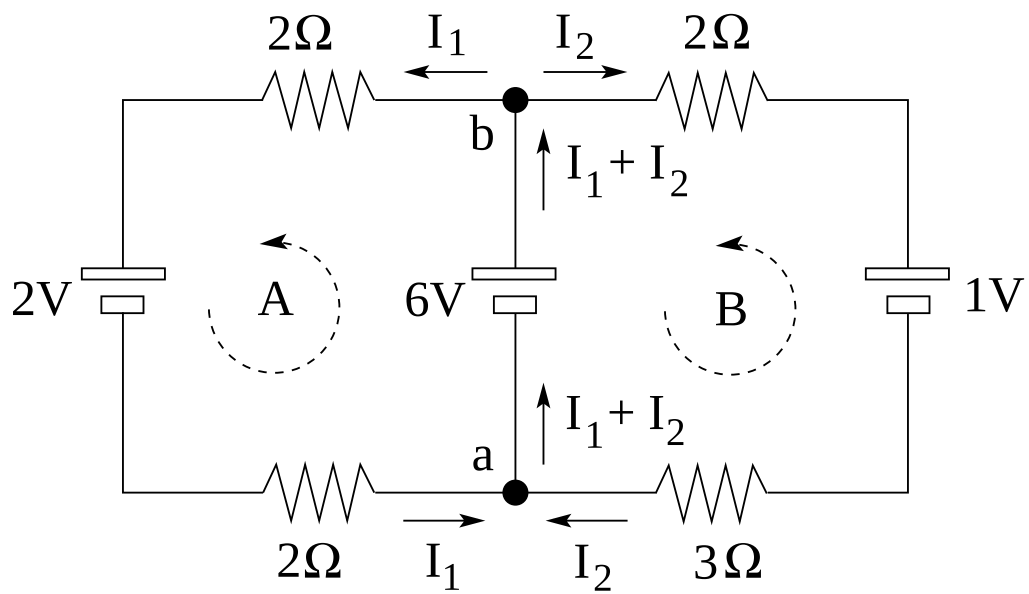
In the circuit of steady currents, use the junction rule to find the unknown currents $I_1, \ldots, I_6$.



# Applying Kirchhoff’s Rules

Consider the circuit shown below.

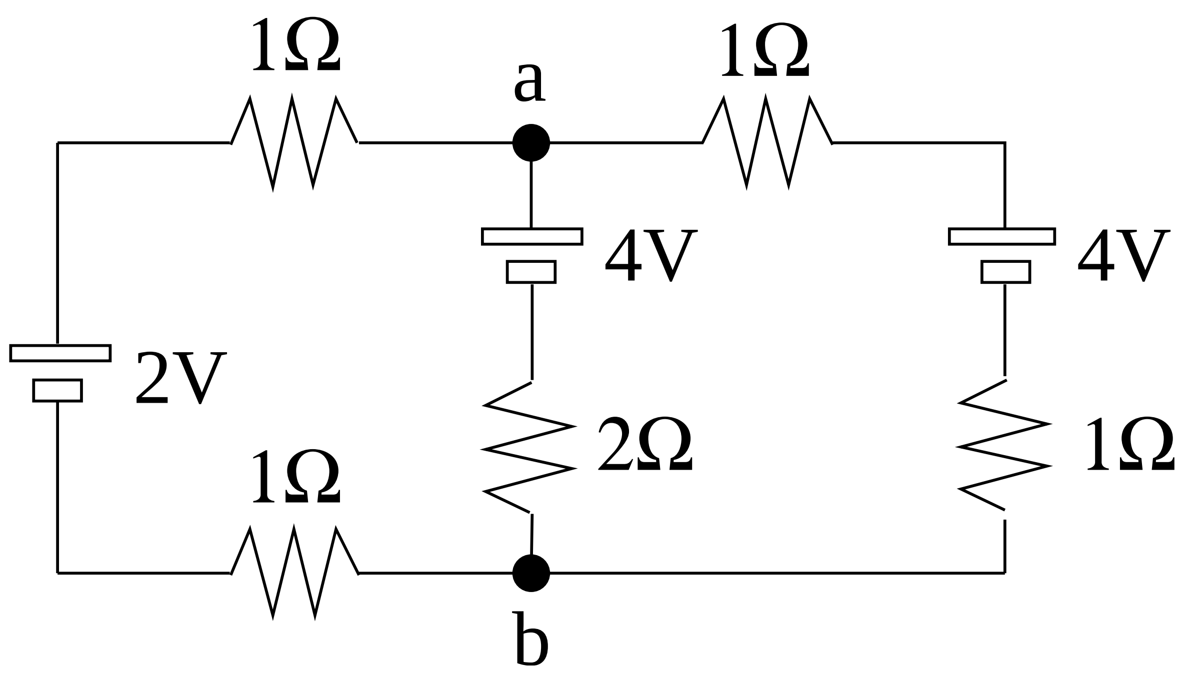
- Junction $a$: $I_1, I_2$ (in); $I_1 + I_2$ (out)
- Junction $b$: $I_1 + I_2$ (in); $I_1, I_2$ (out)
- Two independent currents require the use of two loops.
- Loop $A$ (ccw): $6\text{V} - (2\Omega)I_1 - 2\text{V} - (2\Omega)I_1 = 0$
- Loop $B$ (ccw): $(3\Omega)I_2 + 1\text{V} + (2\Omega)I_2 - 6\text{V} = 0$
- Solution: $I_1 = 1\text{A}, \quad I_2 = 1\text{A}$



# Resistor Circuit (11)

Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.



# Resistor Circuit (9)

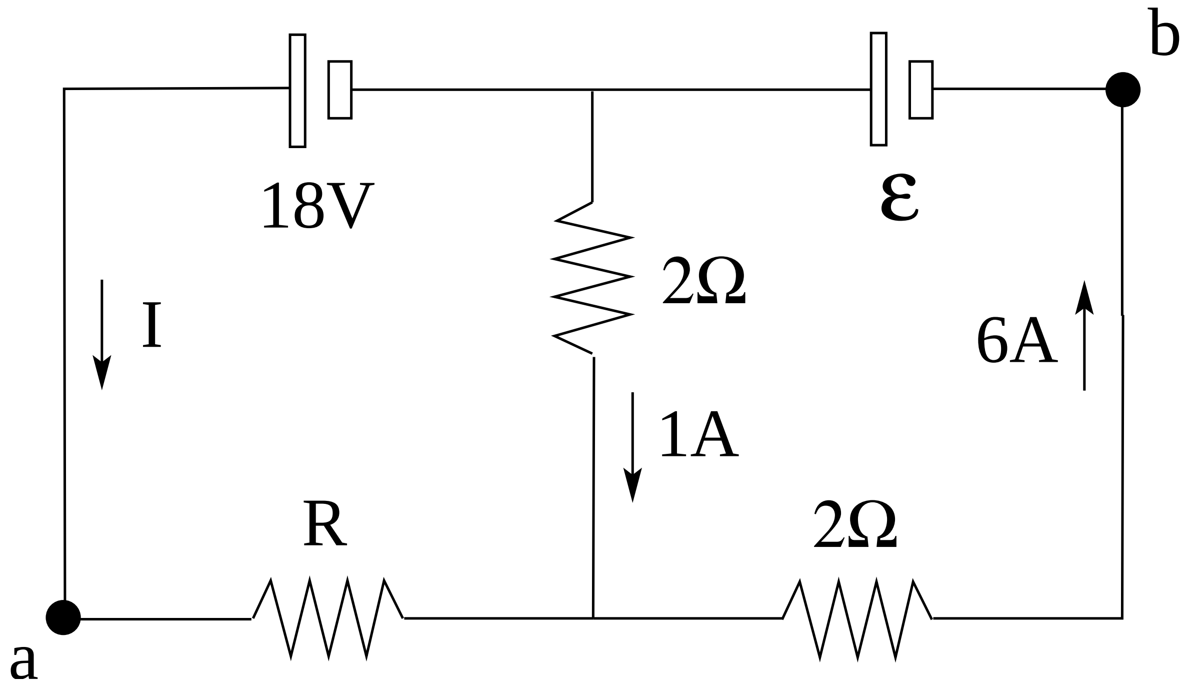
Use Kirchhoff's rules to find

(a) the current $I$,

(b) the resistance $R$,

(c) the emf $\mathcal{E}$,

(d) the voltage $V_{ab} \equiv V_b - V_a$.



# Resistor Circuit (10)

Consider the electric circuit shown.

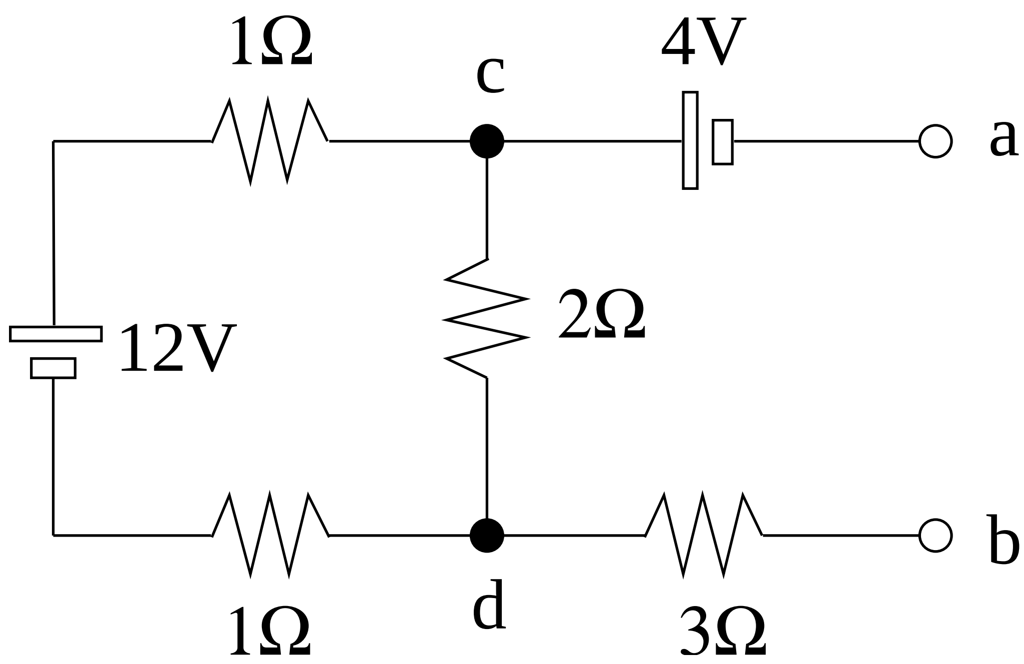
(a) Find the current through the 12V battery.

(b) Find the current through the $2\Omega$ resistor.

(c) Find the total power dissipated.

(d) Find the voltage $V_{cd} \equiv V_d - V_c$.

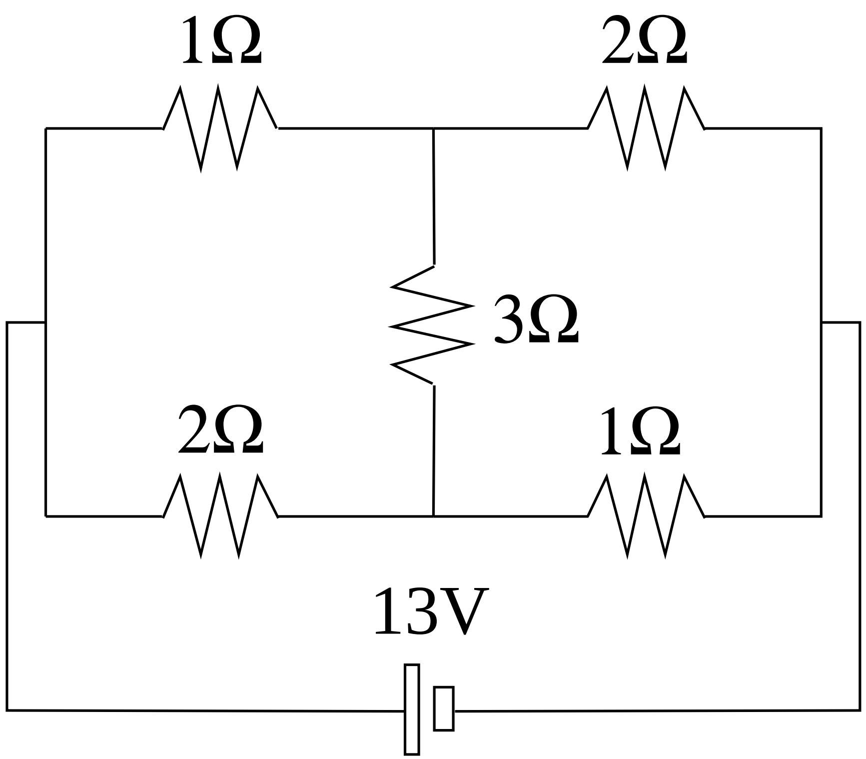
(e) Find the voltage $V_{ab} \equiv V_b - V_a$.



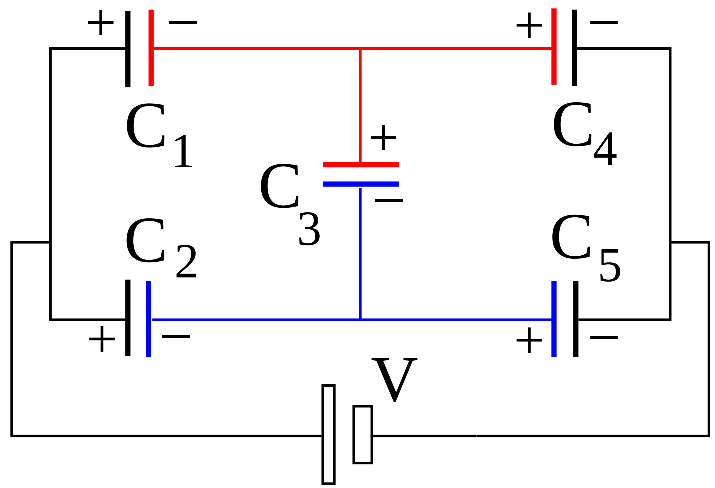
# Resistor Circuit (12)

Consider the electric circuit shown.

- Find the equivalent resistance $R_{eq}$ of the circuit.
- Find the total power $P$ dissipated in the circuit.



# More Complex Capacitor Circuit




No two capacitors are in parallel or in series.
Solution requires different strategy:

- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications: $C_1, \ldots, Q_5, V$.
Five equations for unknowns $Q_1, \ldots, Q_5$:

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\frac{Q_5}{C_5} + \frac{Q_3}{C_3} - \frac{Q_4}{C_4} = 0$
- $\frac{Q_2}{C_2} - \frac{Q_1}{C_1} - \frac{Q_3}{C_3} = 0$
- $V - \frac{Q_1}{C_1} - \frac{Q_4}{C_4} = 0$

Equivalent capacitance: $C_{eq} = \frac{Q_1 + Q_2}{V}$

(a) $C_m = 1\text{pF}$, $m = 1, \ldots, 5$ and $V = 1\text{V}$:

$$C_{eq} = 1\text{pF}, \; Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b) $C_m = m\,\text{pF}$, $m = 1, \ldots, 5$ and $V = 1\text{V}$:

$$C_{eq} = \frac{159}{71}\text{pF}, \; Q_1 = \frac{55}{71}\text{pC}, \; Q_2 = \frac{104}{71}\text{pC},$$

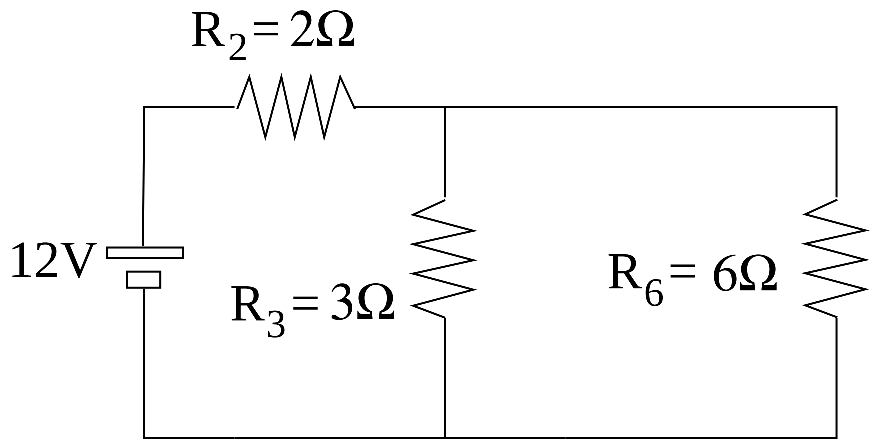
$$Q_3 = -\frac{9}{71}\text{pC}, \; Q_4 = \frac{64}{71}\text{pC}, \; Q_5 = \frac{95}{71}\text{pC}.$$

# Intermediate Exam II: Problem #2 (Spring '05)

Consider the electrical circuit shown.

(a) Find the equivalent resistance $R_{eq}$.

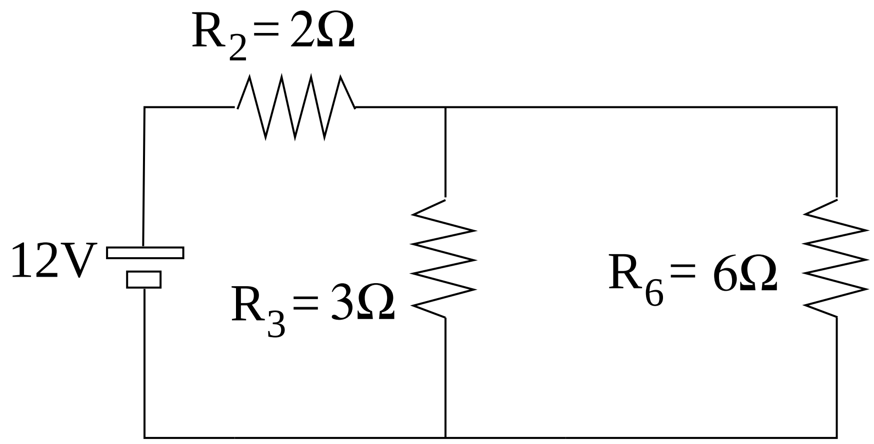
(b) Find the current $I_3$ through resistor $R_3$.



Consider the electrical circuit shown.

(a) Find the equivalent resistance $R_{eq}$.

(b) Find the current $I_3$ through resistor $R_3$.



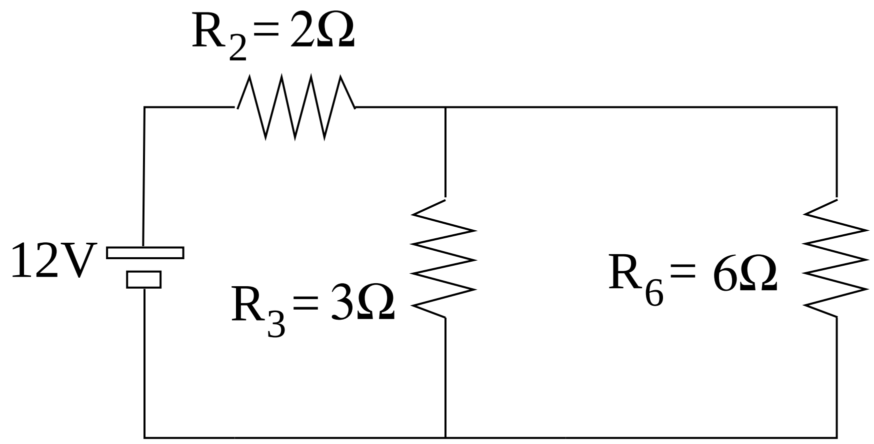

**Solution:**

(a) $R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6}\right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$

Consider the electrical circuit shown.

(a) Find the equivalent resistance $R_{eq}$.

(b) Find the current $I_3$ through resistor $R_3$.




**Solution:**

(a) $R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6}\right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$

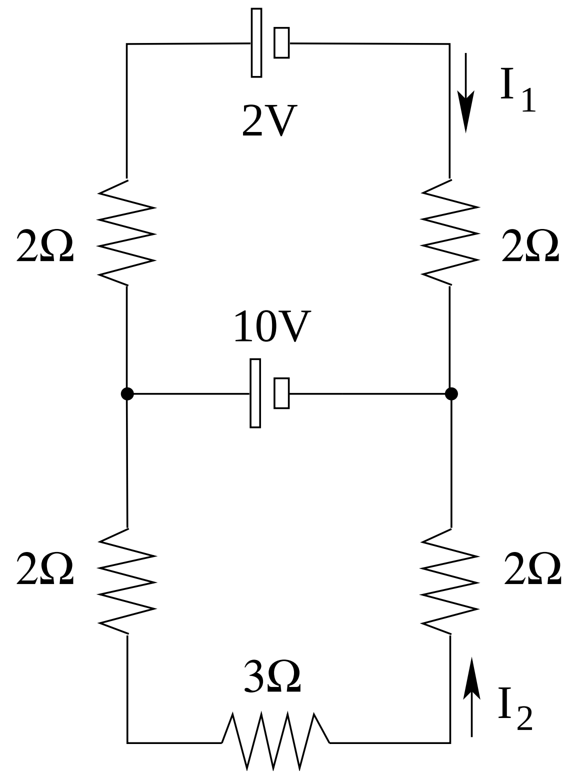
(b) $I_2 = I_{36} = \frac{12\text{V}}{R_{eq}} = 3\text{A}$

$\Rightarrow V_3 = V_{36} = I_{36}R_{36} = 6\text{V} \quad \Rightarrow I_3 = \frac{V_3}{R_3} = 2\text{A}.$

Consider the two-loop circuit shown.

(a) Find the current $I_1$.

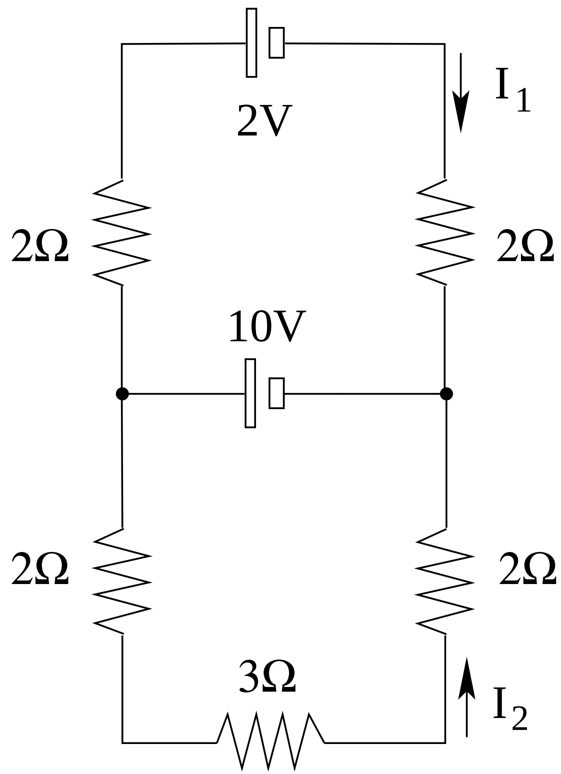
(b) Find the current $I_2$.



Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.




**Solution:**

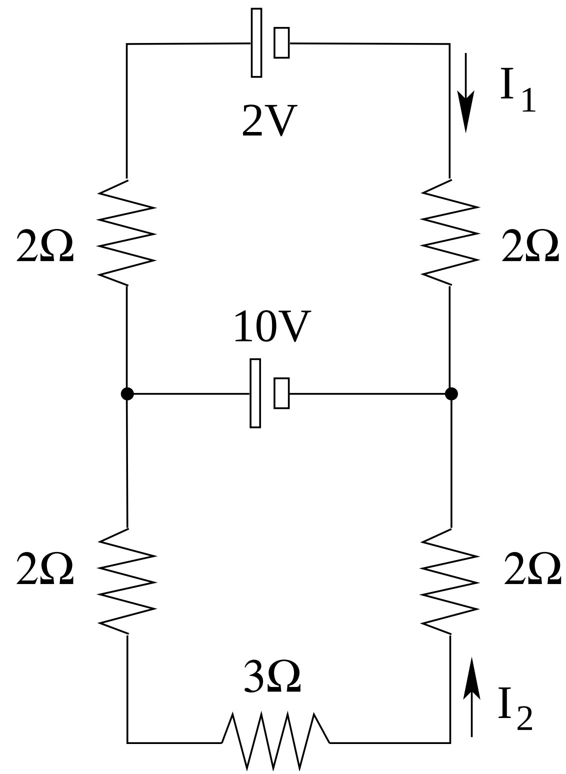
(a) $-(2\Omega)(I_1) + 10\text{V} - (2\Omega)(I_1) - 2\text{V} = 0 \quad \Rightarrow \; I_1 = \dfrac{8\text{V}}{4\Omega} = 2\text{A}.$

# Intermediate Exam II: Problem #2 (Spring '06)

Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.




**Solution:**

(a) $-(2\Omega)(I_1) + 10\text{V} - (2\Omega)(I_1) - 2\text{V} = 0 \quad \Rightarrow I_1 = \dfrac{8\text{V}}{4\Omega} = 2\text{A}.$

(b) $-(2\Omega)(I_2) + 10\text{V} - (2\Omega)(I_2) - (3\Omega)(I_2) = 0 \quad \Rightarrow I_2 = \dfrac{10\text{V}}{7\Omega} = 1.43\text{A}.$

# Unit Exam II: Problem #2 (Spring '07)

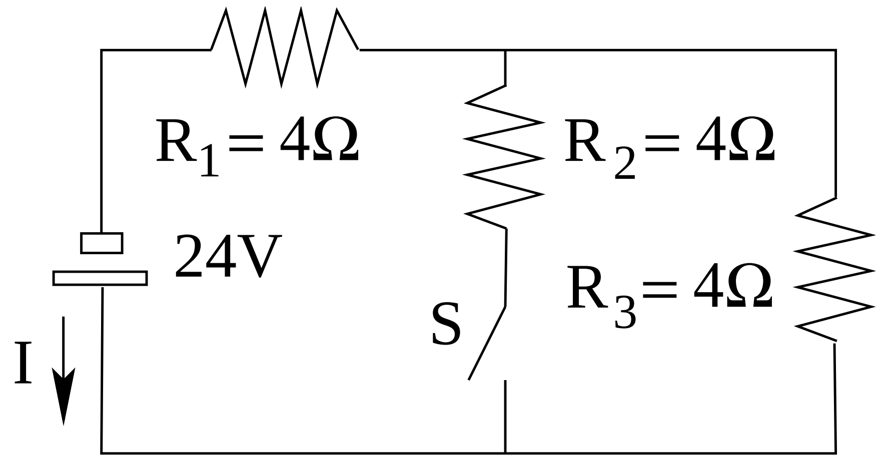
Consider the electric circuit shown.

(a) Find the current $I$ when the switch $S$ is open.

(b) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is open.

(c) Find the current $I$ when the switch $S$ is closed.

(d) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is closed.



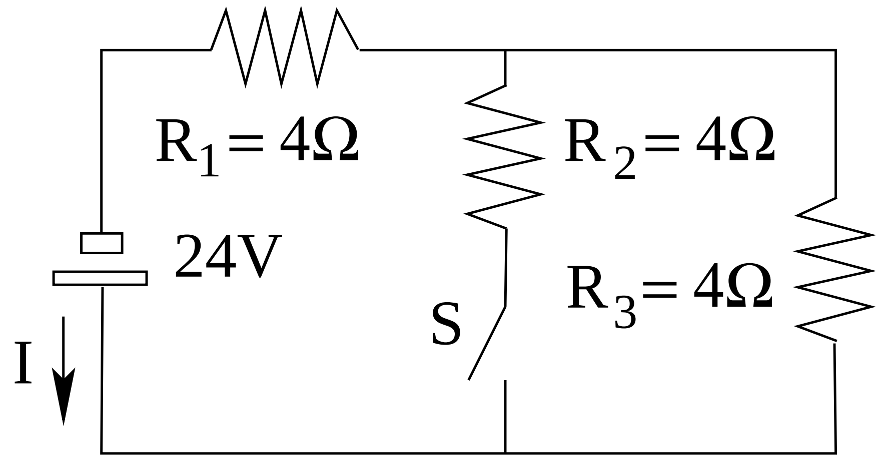
Consider the electric circuit shown.

(a) Find the current $I$ when the switch $S$ is open.

(b) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is open.

(c) Find the current $I$ when the switch $S$ is closed.

(d) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is closed.




**Solution:**

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

Consider the electric circuit shown.

(a) Find the current $I$ when the switch $S$ is open.

(b) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is open.

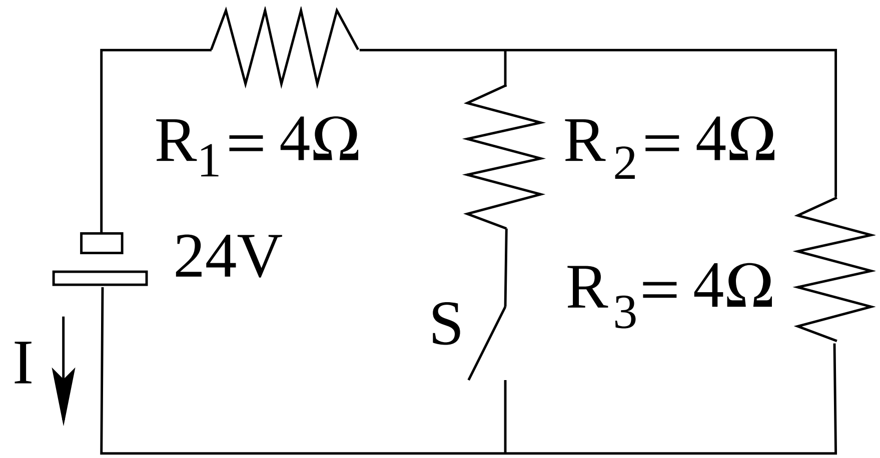
(c) Find the current $I$ when the switch $S$ is closed.

(d) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is closed.

**Solution:**

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

(b) $P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$



# Unit Exam II: Problem #2 (Spring '07)

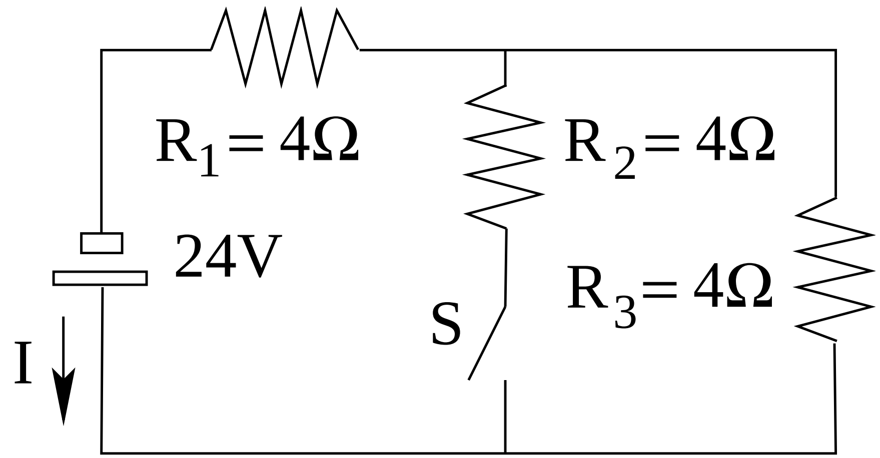
Consider the electric circuit shown.

(a) Find the current $I$ when the switch $S$ is open.

(b) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is open.

(c) Find the current $I$ when the switch $S$ is closed.

(d) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is closed.




**Solution:**

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

(b) $P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$

(c) $I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$

# Unit Exam II: Problem #2 (Spring '07)

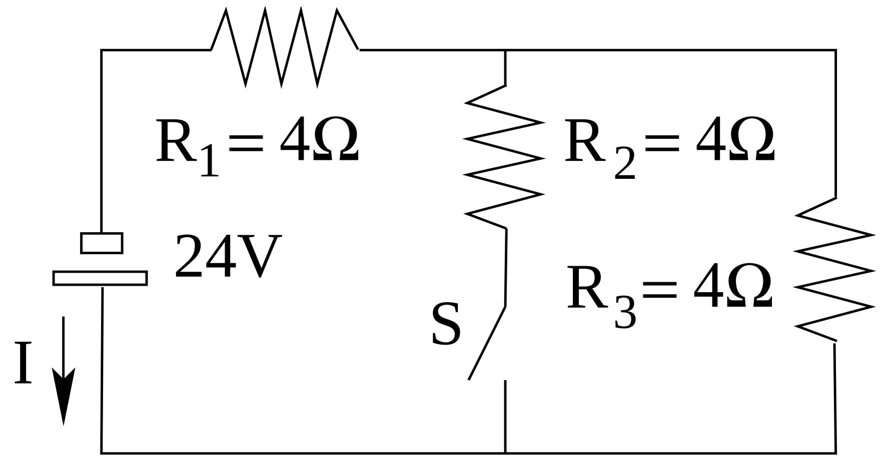
Consider the electric circuit shown.

(a) Find the current $I$ when the switch $S$ is open.

(b) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is open.

(c) Find the current $I$ when the switch $S$ is closed.

(d) Find the power $P_3$ dissipated in resisistor $R_3$ when the switch is closed.




**Solution:**

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

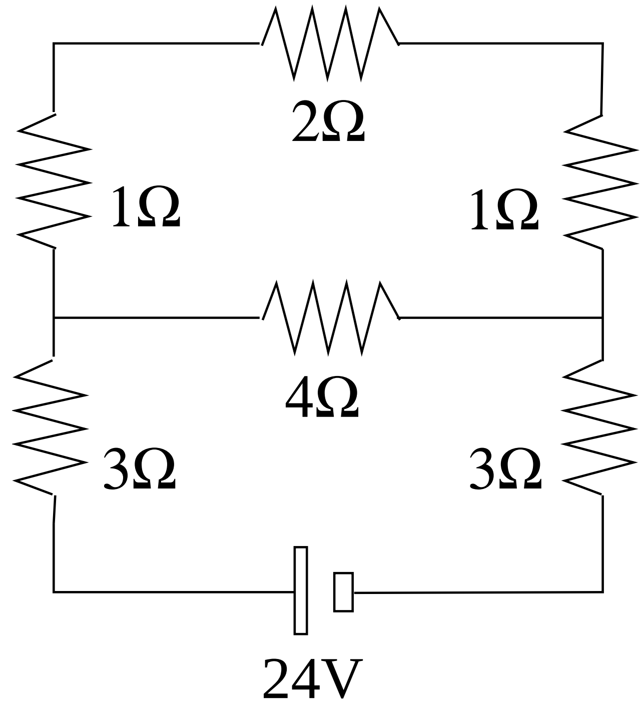
(b) $P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$

(c) $I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$

(d) $P_3 = (2\text{A})^2(4\Omega) = 16\text{W}.$

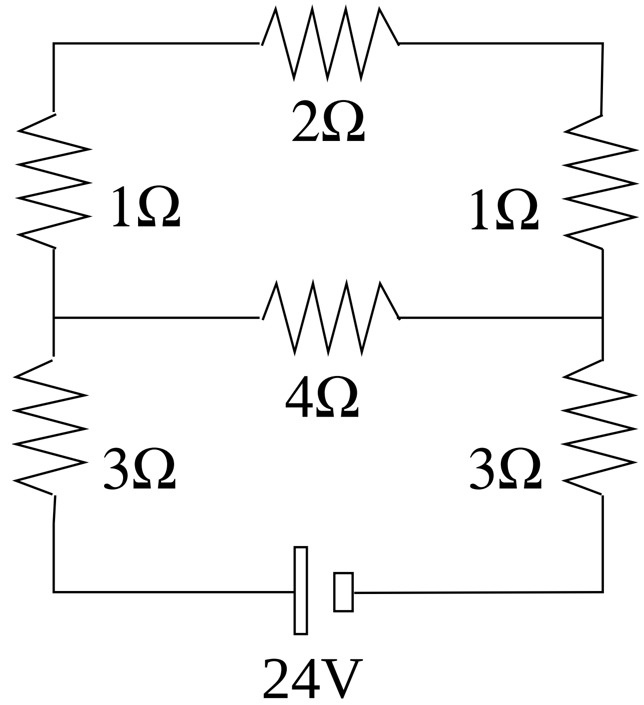
# Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance $R_{eq}$.
(b) Find the power $P$ supplied by the battery.
(c) Find the current $I_4$ through the $4\Omega$-resistor.
(d) Find the voltage $V_2$ across the $2\Omega$-resistor.



# Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance $R_{eq}$.
(b) Find the power $P$ supplied by the battery.
(c) Find the current $I_4$ through the $4\Omega$-resistor.
(d) Find the voltage $V_2$ across the $2\Omega$-resistor.

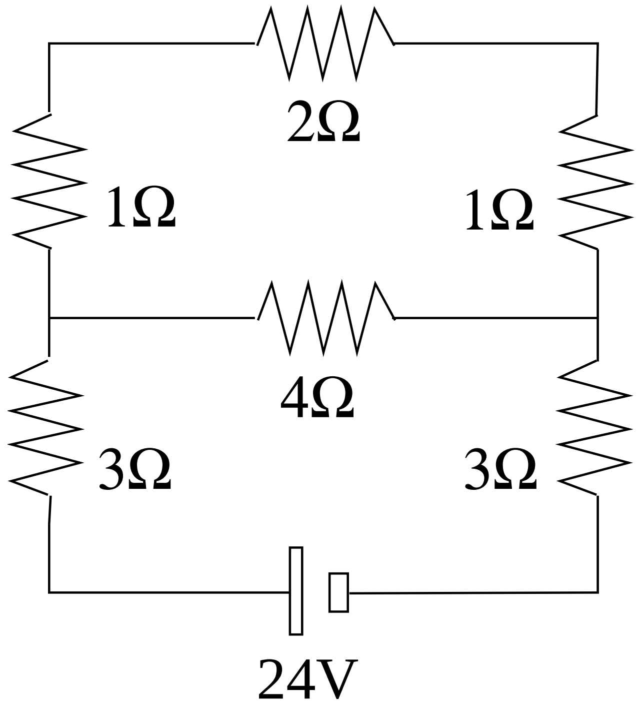



**Solution:**

(a) $R_{eq} = 8\Omega$.

# Unit Exam II: Problem #2 (Spring ’09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance $R_{eq}$.
(b) Find the power $P$ supplied by the battery.
(c) Find the current $I_4$ through the $4\Omega$-resistor.
(d) Find the voltage $V_2$ across the $2\Omega$-resistor.



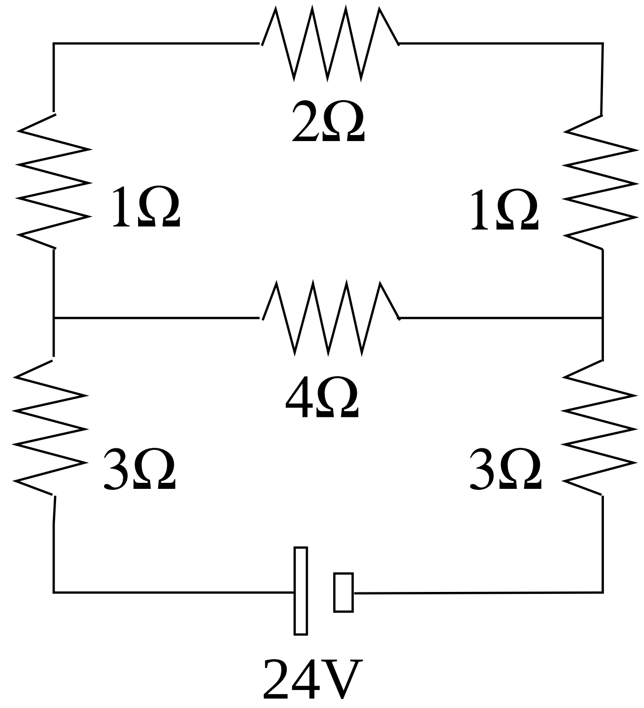

**Solution:**

(a) $R_{eq} = 8\Omega.$

(b) $P = \dfrac{(24\text{V})^2}{8\Omega} = 72\text{W}.$

# Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance $R_{eq}$.
(b) Find the power $P$ supplied by the battery.
(c) Find the current $I_4$ through the $4\Omega$-resistor.
(d) Find the voltage $V_2$ across the $2\Omega$-resistor.




**Solution:**

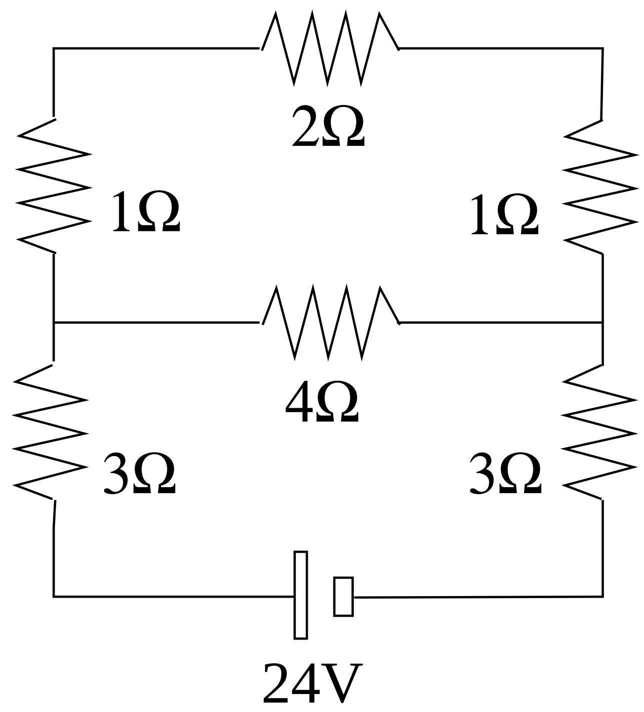
(a) $R_{eq} = 8\Omega.$

(b) $P = \dfrac{(24\text{V})^2}{8\Omega} = 72\text{W}.$

(c) $I_4 = \dfrac{1}{2}\dfrac{24\text{V}}{8\Omega} = 1.5\text{A}.$

# Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance $R_{eq}$.
(b) Find the power $P$ supplied by the battery.
(c) Find the current $I_4$ through the $4\Omega$-resistor.
(d) Find the voltage $V_2$ across the $2\Omega$-resistor.




**Solution:**

(a) $R_{eq} = 8\Omega.$

(b) $P = \dfrac{(24\text{V})^2}{8\Omega} = 72\text{W}.$

(c) $I_4 = \dfrac{1}{2}\dfrac{24\text{V}}{8\Omega} = 1.5\text{A}.$

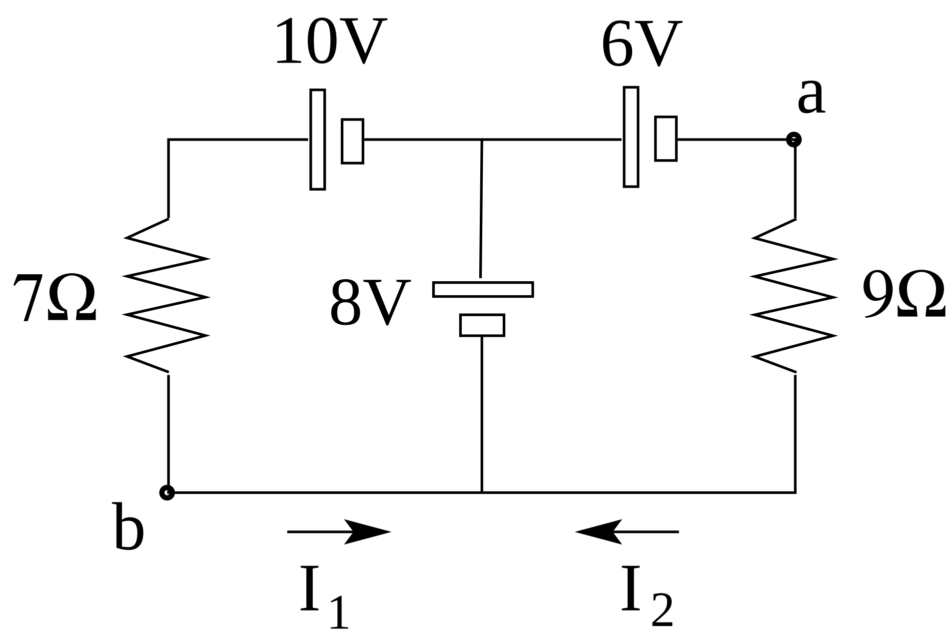
(d) $V_2 = (1.5\text{A})(2\Omega) = 3\text{V}.$

Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.

(c) Find the potential difference $V_a - V_b$.

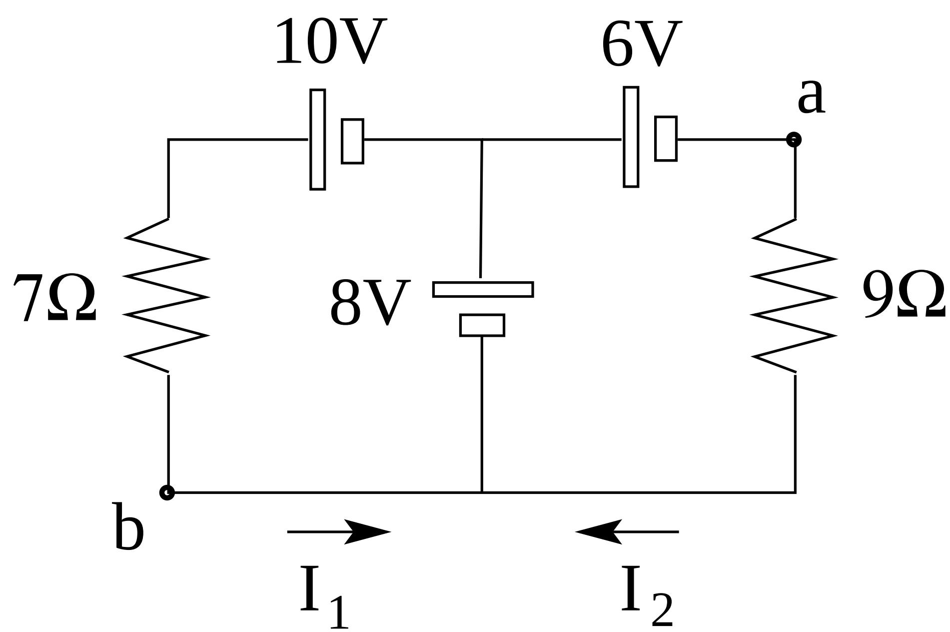


Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.

(c) Find the potential difference $V_a - V_b$.




**Solution:**

(a) $I_1 = \frac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

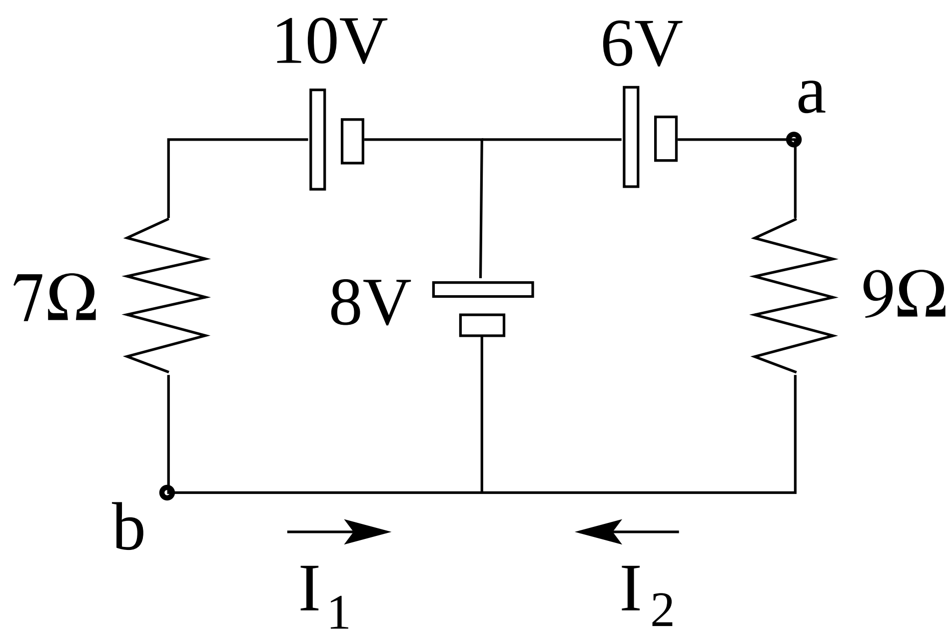
# Unit Exam II: Problem #3 (Spring '07)

Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.

(c) Find the potential difference $V_a - V_b$.




**Solution:**

(a) $I_1 = \dfrac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

(b) $I_2 = \dfrac{8\text{V} - 6\text{V}}{9\Omega} = 0.22\text{A}.$

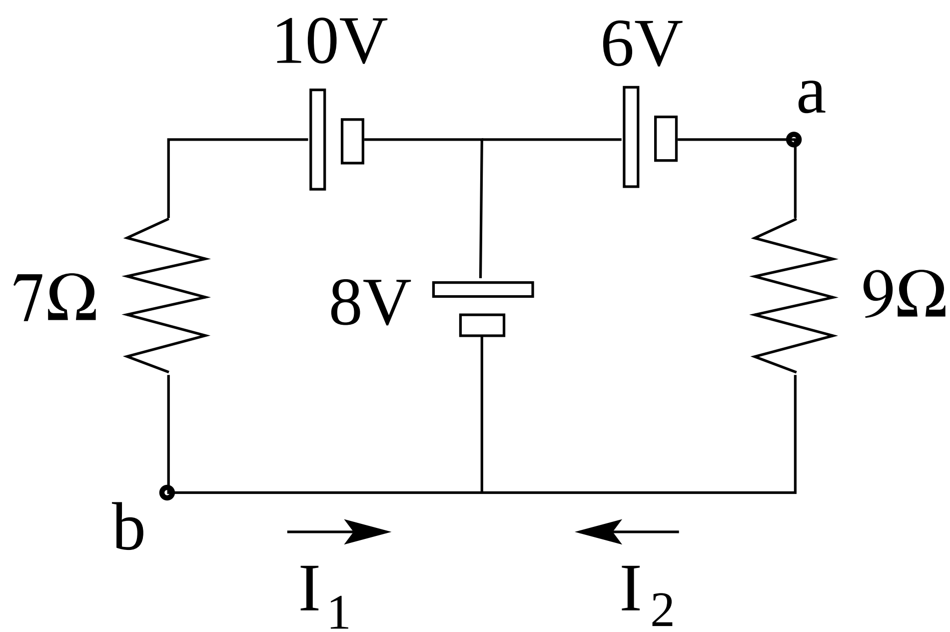
# Unit Exam II: Problem #3 (Spring ’07)

Consider the two-loop circuit shown.

(a) Find the current $I_1$.

(b) Find the current $I_2$.

(c) Find the potential difference $V_a - V_b$.




**Solution:**

(a) $I_1 = \dfrac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

(b) $I_2 = \dfrac{8\text{V} - 6\text{V}}{9\Omega} = 0.22\text{A}.$

(c) $V_a - V_b = 8\text{V} - 6\text{V} = 2\text{V}.$