RC Circuit: Fundamentals

Specifications:
- \( \mathcal{E} \) (emf)
- \( R \) (resistance)
- \( C \) (capacitance)

Switch \( S \):
- a: charging
- b: discharging

Time-dependent quantities:
- \( Q(t) \): instantaneous charge on capacitor
- \( I(t) = \frac{dQ}{dt} \): instantaneous current
- \( V_R(t) = I(t)R \): instantaneous voltage across resistor
- \( V_C(t) = \frac{Q(t)}{C} \): instantaneous voltage across capacitor
RC Circuit: Charging the Capacitor

- Loop rule: \( \mathcal{E} - IR - \frac{Q}{C} = 0 \)

- Differential equation: \( R \frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E} \Rightarrow \frac{dQ}{dt} = \frac{\mathcal{E}C - Q}{RC} \)

\[
\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln \left( \frac{\mathcal{E}C - Q}{\mathcal{E}C} \right) = \frac{t}{RC} \Rightarrow \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}
\]

- Charge on capacitor: \( Q(t) = \mathcal{E}C \left[ 1 - e^{-t/RC} \right] \)

- Current through resistor: \( I(t) \equiv \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} \)
RC Circuit: Discharging the Capacitor

- Loop rule: \( IR + \frac{Q}{C} = 0 \)

- Differential equation: \( R \frac{dQ}{dt} + \frac{Q}{C} = 0 \) \( \Rightarrow \frac{dQ}{dt} = -\frac{Q}{RC} \)
  \[ \int_{\mathcal{E}C}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{dt}{RC} \Rightarrow \ln \left( \frac{Q}{\mathcal{E}C} \right) = -\frac{t}{RC} \Rightarrow \frac{Q}{\mathcal{E}C} = e^{-t/RC} \]

- Charge on capacitor: \( Q(t) = \mathcal{E}Ce^{-t/RC} \)

- Current through resistor: \( I(t) \equiv \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/RC} \)
Loop rule:  \[ IR + \frac{Q}{C} = \mathcal{E} \quad (I \text{ is positive}) \]

- \( I \mathcal{E} \): rate at which emf source delivers energy
- \( IV_R = I^2 R \): rate at which energy is dissipated in resistor
- \( IV_C = \frac{IQ}{C} \): rate at which energy is stored in capacitor

Balance of energy transfer:  \[ I^2 R + \frac{IQ}{C} = I \mathcal{E} \]
Loop rule: \( IR + \frac{Q}{C} = 0 \)  
\( (I \text{ is negative}) \)

- \( IV_R = I^2 R \): rate at which energy is dissipated in resistor
- \( IV_C = \frac{IQ}{C} \): rate at which capacitor releases energy

Balance of energy transfer:  \( I^2 R + \frac{IQ}{C} = 0 \)
Speciﬁcation of \textit{RC} circuit by 3 device properties:

\begin{itemize}
  \item $\mathcal{E}$ [V] (emf)
  \item $R$ [Ω] (resistance)
  \item $C$ [F] (capacitance)
\end{itemize}

Physical properties of \textit{RC} circuit during charging process determined by 3 combinations of the device properties:

\begin{itemize}
  \item $\mathcal{E}/R = I(t = 0)$: rate at which current flows onto capacitor initially
  \item $\mathcal{E}C = Q(t = \infty)$: total charge placed on capacitor ultimately
  \item $RC = \tau$: time it takes to place 63\% of the charge onto the capacitor
    \[ 1 - e^{-1} = 0.632 \ldots \] 
\end{itemize}
This circuit has been running for a very long time.

(a) Find the current through the 18V battery.
(b) Find the total power dissipated in the resistors.
(c) Find the charge stored on the capacitor.
The switches are closed at $t = 0$. This begins the charging process in each $RC$ circuit.

Name the circuit in which...

(i) the charge flows into the capacitor at the highest rate initially,
(ii) the capacitor has the most charge ultimately,
(iii) the capacitor is 63% full in the shortest time.
At time $t = 0$ the capacitor in this circuit is discharged and the switch is being closed.

Find the current $I_1$
(a) at $t = 0$,
(b) at $t = \infty$.

Find the current $I_2$
(c) at $t = 0$,
(d) at $t = \infty$. 

12V

[Diagram of the RC circuit with 5nF, 2Ω, and 2Ω resistors connected to a 12V source and an open switch.]
In this 3-loop $RC$ circuit, the switch $S$ is closed at time $t = 0$.

(a) Find the currents $I_1, I_2, I_3$ just after the switch has been closed.
(b) Find the currents $I_1, I_2, I_3$ a very long time later.
In the $RC$ circuit shown, the switch $S$ has been open for a long time.

(a) Find the currents $I_1$ and $I_2$ immediately after the switch has been closed.
(b) Find the currents $I_1$ and $I_2$ a very long time later.

![Diagram of RC circuit with components 2Ω, 3 µF, 4Ω, and 24V connected through switch S and currents $I_1$ and $I_2$.]
In the $RC$ circuit shown, both switches are initially open and the capacitor is discharged.

(a) Close switch $S_1$ and find the currents $I_1$ and $I_2$ immediately afterwards.
(b) Find the currents $I_1 I_2$ and the charge $Q$ on the capacitor a very long time later.
(c) Now close switch $S_2$ also and find the currents $I_1$ and $I_2$ immediately afterwards.
(d) Find the currents $I_1, I_2$ and the charge $Q$ on the capacitor a very long time later.
In the \( RC \) circuit shown, the switch has been open for a long time.

Find the currents \( I_1, I_2, I_3 \) and the charge \( Q \) on the capacitor

(a) right after the switch has been closed,
(b) a very long time later.
The circuit shown is that of a flashing lamp, such as are attached to barrels at highway construction sites. The power is supplied by a battery with $\mathcal{E} = 95\,\text{V}$. The fluorescent lamp $L$ is connected in parallel to the capacitor with $C = 0.15\,\mu\text{F}$ of an $RC$ circuit. Current passes through the lamp only when the potential difference across it reaches the breakdown voltage $V_L = 72\,\text{V}$. In this event, the capacitor discharges through the lamp, and the lamp flashes briefly.

Suppose that two flashes per second are needed. What should the resistance $R$ be?
The circuit shown contains two identical capacitors and two ideal diodes. A 100V battery is connected to the two input terminals \( a \) and \( b \). Find the voltage at the output terminals \( c \) and \( d \)

(1) if \( a \) is the positive input terminal,

(2) if \( b \) is the positive input terminal.

Note: An ideal diode is a perfect one-way street for electric currents. It lets a current through unimpeded in the direction of the arrow and totally blocks any current in the opposite direction.
Consider the $RC$ circuit shown. The switch has been closed for a long time.
(a) Find the current $I_B$ flowing through the battery.
(b) Find the voltage $V_C$ across the capacitor.
(c) Find the charge $Q$ on the capacitor.
(d) Find the current $I_3$ flowing through the $3\Omega$-resistor right after the switch has been opened.
This $RC$ circuit has been running for a long time.

(a) Find the current $I_2$ through the resistor $R_2$.
(b) Find the voltage $V_C$ across the capacitor.
This $RC$ circuit has been running for a long time.

(a) Find the current $I_2$ through the resistor $R_2$.
(b) Find the voltage $V_C$ across the capacitor.

Solution:

(a) $I_C = 0$, 
$$ I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{12\text{V}}{6\Omega} = 2\text{A}. $$
This $RC$ circuit has been running for a long time.

(a) Find the current $I_2$ through the resistor $R_2$.
(b) Find the voltage $V_C$ across the capacitor.

Solution:

(a) $I_C = 0$, $I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{12\text{V}}{6\Omega} = 2\text{A}$.

(b) $V_C = V_2 = I_2 R_2 = (2\text{A})(4\Omega) = 8\text{V}$. 