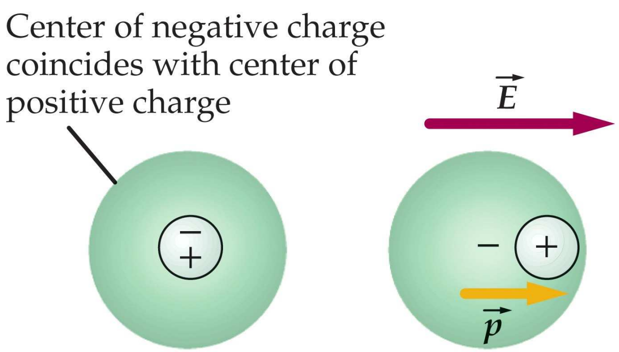
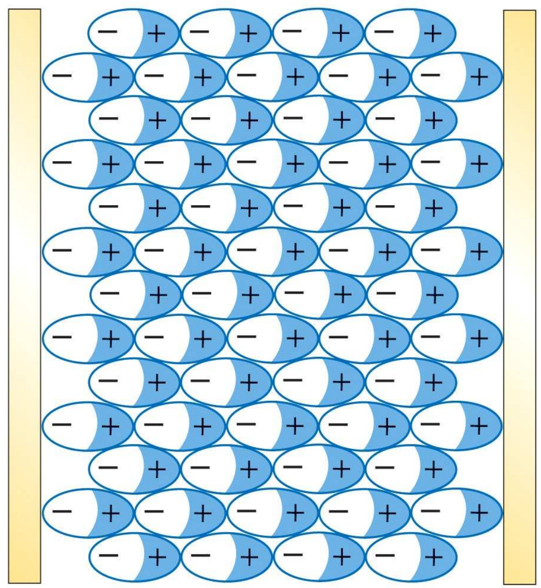
Capacitor with Dielectric

Most capacitors have a dielectric (insulating solid or liquid material) in the space between the conductors. This has several advantages:

- Physical separation of the conductors.
- Prevention of dielectric breakdown.
- Enhancement of capacitance.

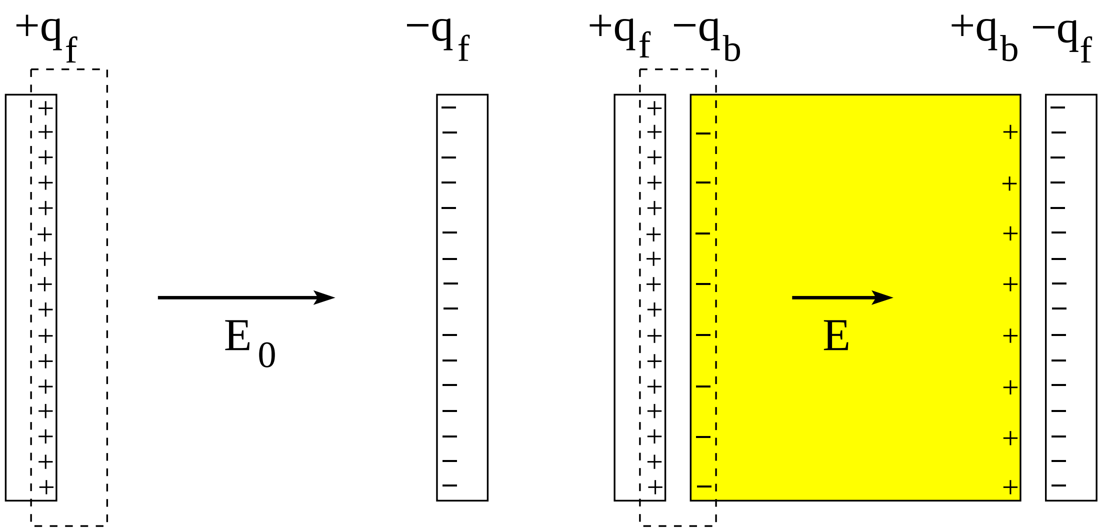
The dielectric is polarized by the electric field between the capacitor plates.





Parallel-Plate Capacitor with Dielectric (1)

The polarization produces a bound charge on the surface of the dielectric.

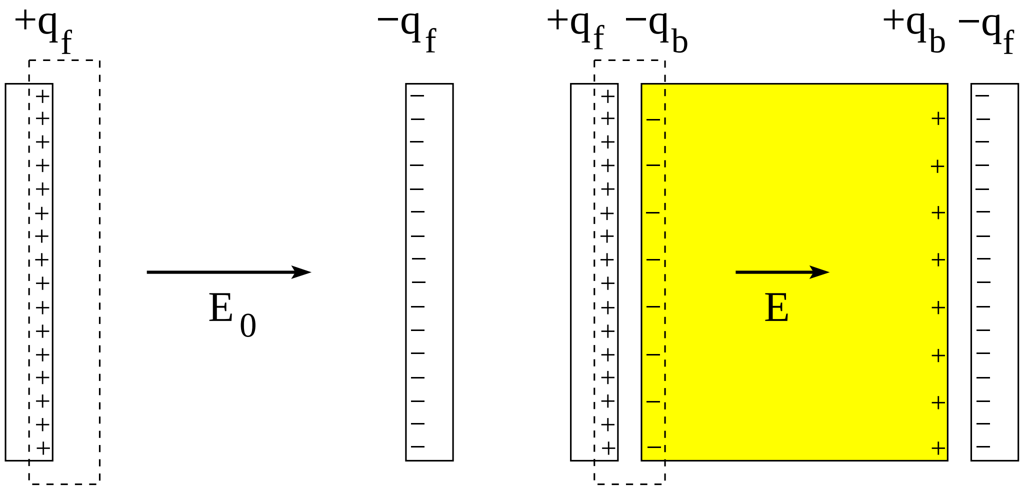


The bound surface charge has the effect of reducing the electric field between the plates from $\vec{E}_0$ to $\vec{E}$.

- A: area of plates
- d: separation between plates
- $\pm q_f$: free charge on plate
- $\pm q_b$: bound charge on surface of dielectric
- $\vec{E}_0$: electric field in vacuum
- $\vec{E}$: electric field in dielectric

Parallel-Plate Capacitor with Dielectric (2)

Use Gauss' law to determine the electric fields $\vec{E}_0$ and $\vec{E}$.



- Field in vacuum: $E_0 A = \dfrac{q_f}{\epsilon_0} \quad \Rightarrow \quad E_0 = \dfrac{q_f}{\epsilon_0 A}$
- Field in dielectric: $EA = \dfrac{q_f - q_b}{\epsilon_0} \quad \Rightarrow \quad E = \dfrac{q_f - q_b}{\epsilon_0 A} < E_0$
- Voltage: $V_0 = E_0 d$ (vacuum), $V = Ed = \dfrac{V_0}{\kappa} < V_0$ (dielectric)

Dielectric constant: $\kappa \equiv \dfrac{E_0}{E} = \dfrac{q_f}{q_f - q_b} > 1.$ Permittivity of dielectric: $\epsilon = \kappa\epsilon_0$.

Dielectric Materials

TABLE 24-1

Dielectric Constants and Dielectric Strengths of Various Materials

Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

- Dielectrics increase the capacitance: $C/C_0 = \kappa$.
- The capacitor is discharged spontaneously across the dielectric if the electric field exceeds the value quoted as dielectric strength.

Impact of Dielectric (1)

What happens when a dielectric is placed into a capacitor
with the **charge on the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = Q_0$
electric field	E_0	$E = \dfrac{E_0}{\kappa} < E_0$
voltage	V_0	$V = \dfrac{V_0}{\kappa} < V_0$
capacitance	$C_0 = \dfrac{Q_0}{V_0}$	$C = \dfrac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \dfrac{Q_0^2}{2C_0}$	$U = \dfrac{Q^2}{2C} = \dfrac{U_0}{\kappa} < U_0$
energy density	$u_E^{(0)} = \dfrac{1}{2}\epsilon_0 E_0^2$	$u_E = \dfrac{u_E^{(0)}}{\kappa} = \dfrac{1}{2}\kappa\epsilon_0 E^2 < u_E^{(0)}$

Impact of Dielectric (2)

What happens when a dielectric is placed into a capacitor
with the **voltage across the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = \kappa Q_0$
electric field	E_0	$E = E_0$
voltage	V_0	$V = V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{1}{2}C_0 V_0^2$	$U = \frac{1}{2}CV^2 = \kappa U_0 > U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \kappa u_E^{(0)} = \frac{1}{2}\kappa\epsilon_0 E^2 > u_E^{(0)}$

Stacked Dielectrics

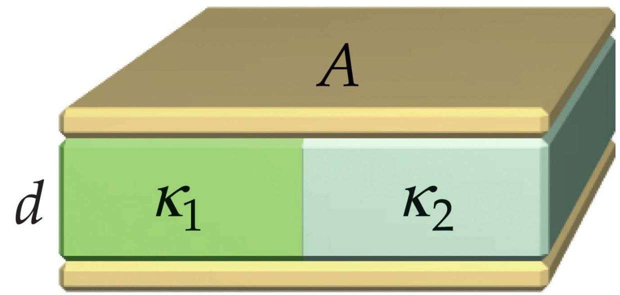
Consider a parallel-plate capacitor with area A of each plate and spacing d.

- Capacitance without dielectric: $C_0 = \dfrac{\epsilon_0 A}{d}$.

- Dielectrics stacked in parallel: $C = C_1 + C_2$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A/2}{d},\ C_2 = \kappa_2 \epsilon_0 \dfrac{A/2}{d}$.

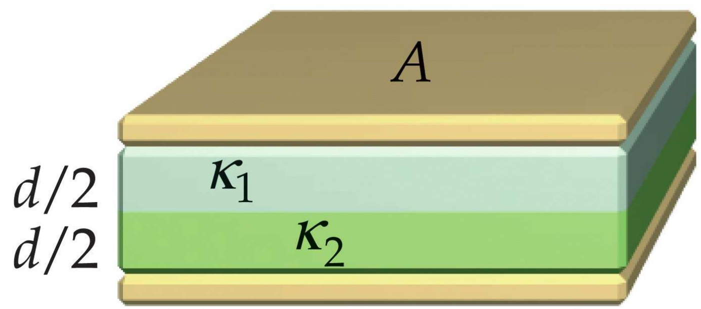
  $\Rightarrow\ C = \dfrac{1}{2}(\kappa_1 + \kappa_2) C_0$.



- Dielectrics stacked in series: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A}{d/2},\ C_2 = \kappa_2 \epsilon_0 \dfrac{A}{d/2}$

  $\Rightarrow\ C = \dfrac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} C_0$.



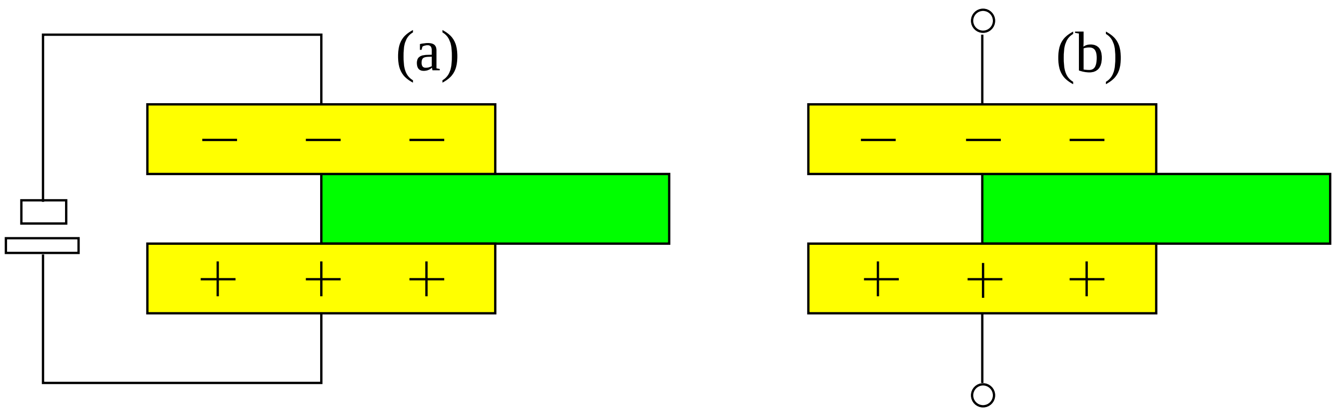
Lateral Force on Dielectric

Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place
at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place
at **constant charge** on the plates.

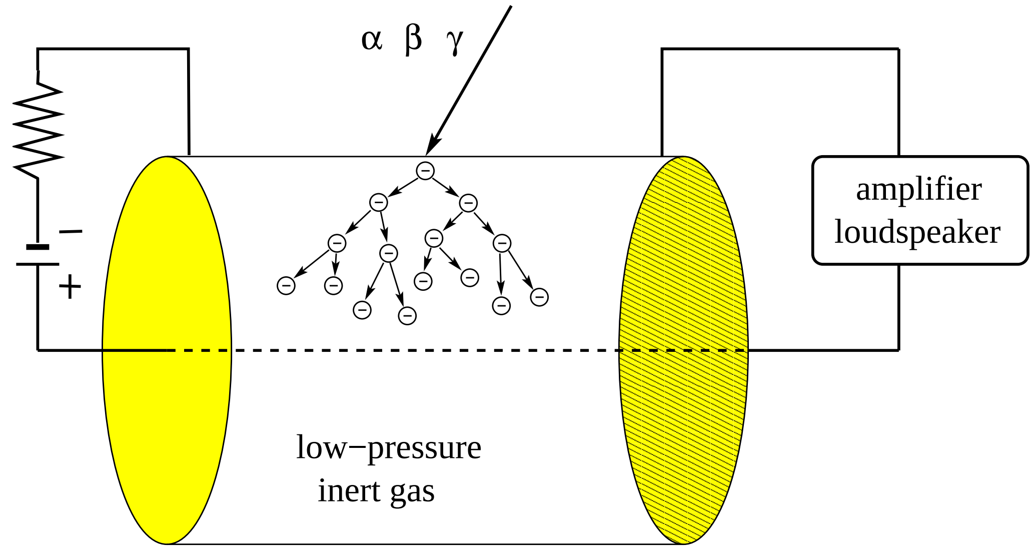
Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.



Geiger Counter

Radioactive atomic nuclei produce high-energy particles of three different kinds:

- α-particles are ^{4}He nuclei.
- β-particles are electrons or positrons.
- γ-particles are high-energy photons.



- Free electrons produced by ionizing radiation are strongly accelerated toward the central wire.
- Collisions with gas atoms produce further free electrons, which are accelerated in the same direction.
- An avalanche of electrons reaching the wire produces a current pulse in the circuit.

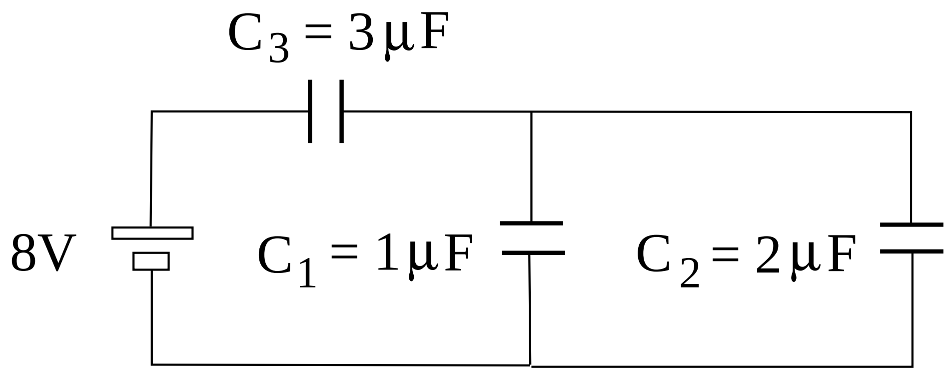
Intermediate Exam II: Problem #1 (Spring ’05)

The circuit of capacitors connected to a battery is at equilibrium.

(a) Find the equivalent capacitance C_{eq}.

(b) Find the voltage V_3 across capacitor C_3.

(c) Find the the charge Q_2 on capacitor C_2.



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(a) Find the equivalent capacitance C_{eq}.

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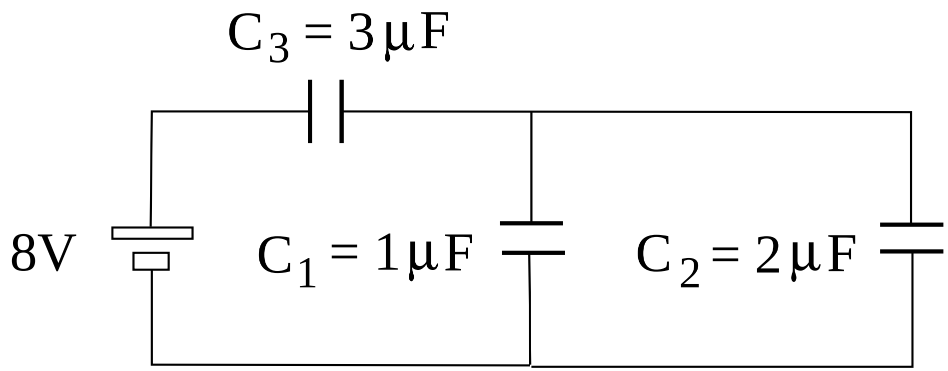


Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\text{F}.$

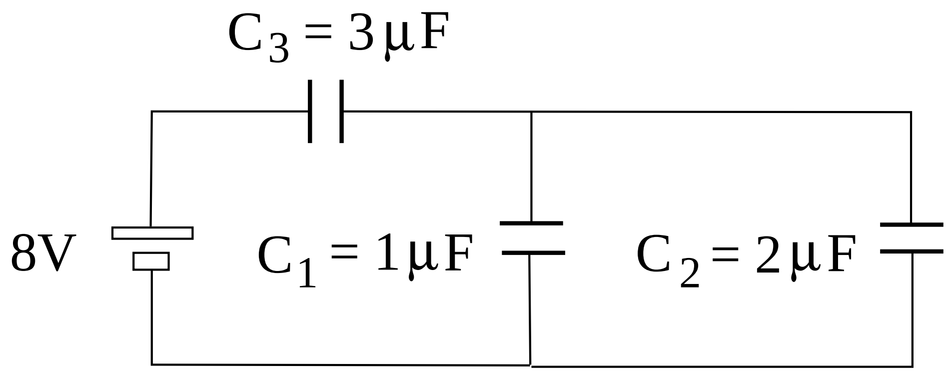
Intermediate Exam II: Problem #1 (Spring '05)

The circuit of capacitors connected to a battery is at equilibrium.

(a) Find the equivalent capacitance C_{eq}.

(b) Find the voltage V_3 across capacitor C_3.

(c) Find the the charge Q_2 on capacitor C_2.



Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\text{F}.$

(b) $Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\text{V}) = 12\mu\text{C}$

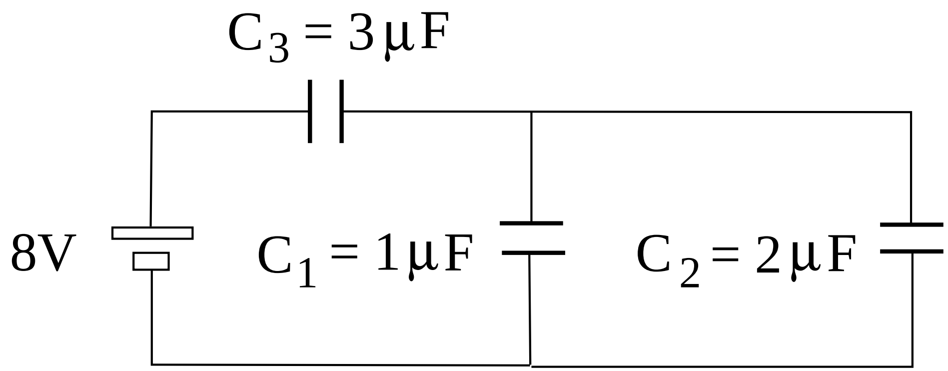
$\Rightarrow V_3 = \frac{Q_3}{C_3} = \frac{12\mu\text{C}}{3\mu\text{F}} = 4\text{V}.$

The circuit of capacitors connected to a battery is at equilibrium.

(a) Find the equivalent capacitance C_{eq}.

(b) Find the voltage V_3 across capacitor C_3.

(c) Find the the charge Q_2 on capacitor C_2.



Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\text{F}.$

(b) $Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\text{V}) = 12\mu\text{C}$

$\Rightarrow V_3 = \frac{Q_3}{C_3} = \frac{12\mu\text{C}}{3\mu\text{F}} = 4\text{V}.$

(c) $Q_2 = V_2 C_2 = 8\mu\text{C}.$

Unit Exam II: Problem #1 (Spring '07)

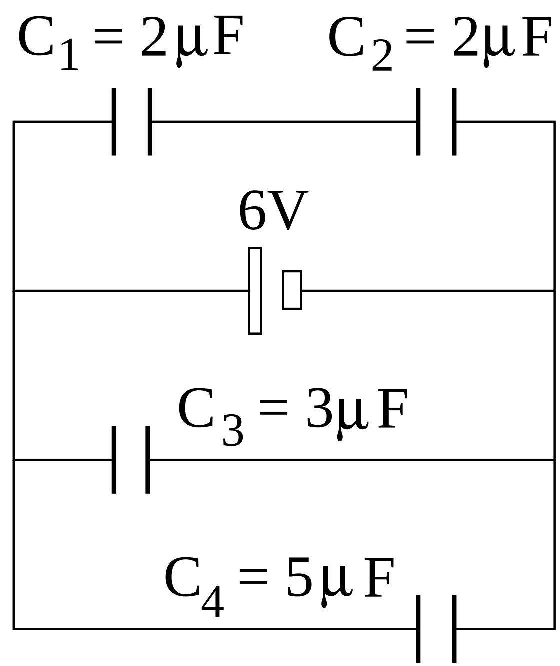
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Unit Exam II: Problem #1 (Spring '07)

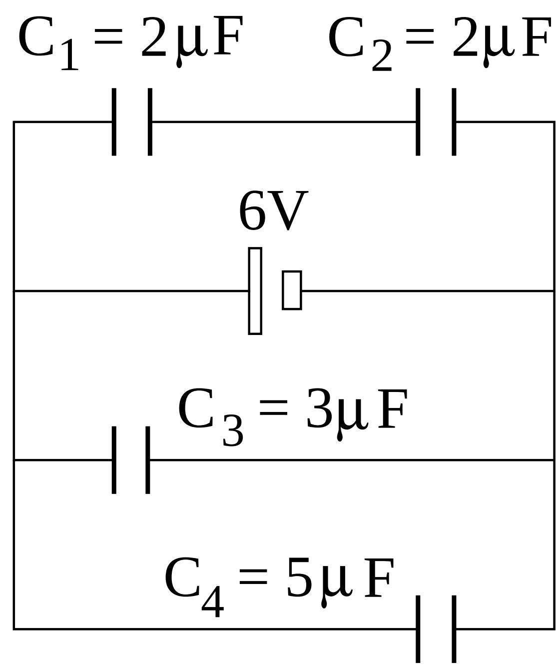
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}.$

Unit Exam II: Problem #1 (Spring '07)

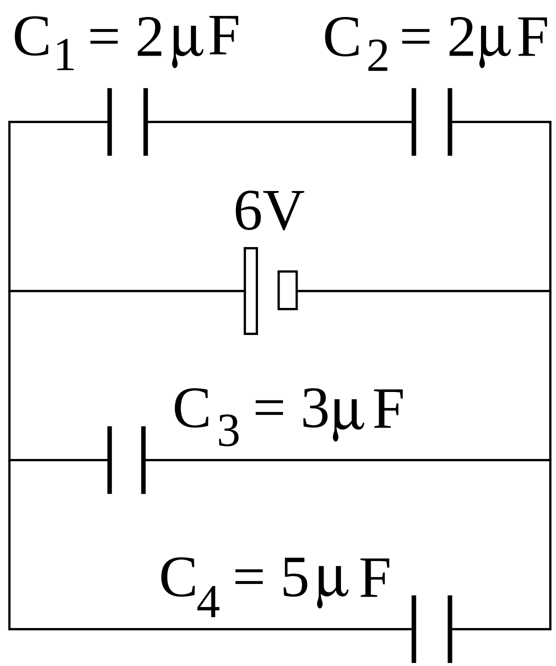
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}$.

(b) $V_4 = 6\text{V}$.

Unit Exam II: Problem #1 (Spring '07)

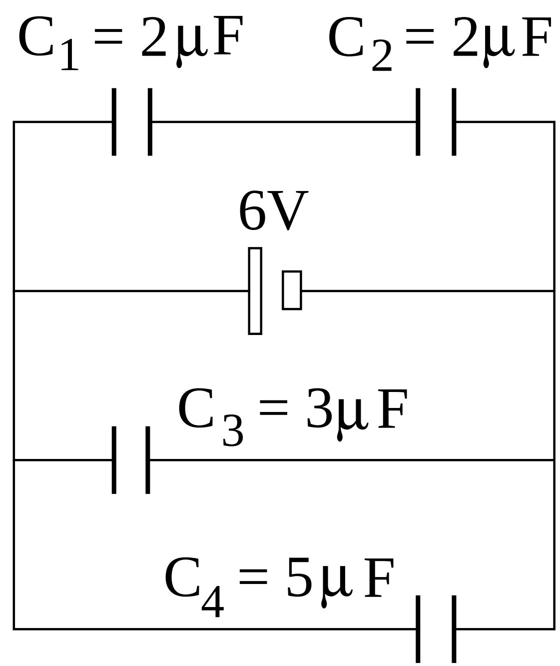
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}$.

(b) $V_4 = 6\text{V}$.

(c) $V_2 = \frac{1}{2}6\text{V} = 3\text{V}$.

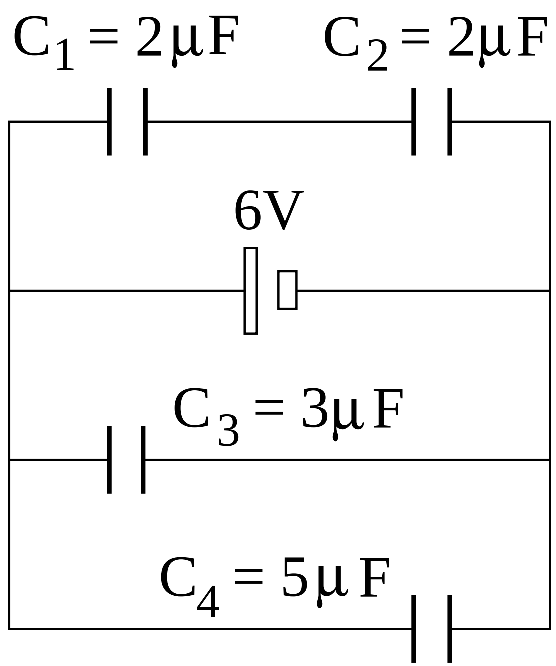
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\mathrm{F})(6\mathrm{V})^2 = 54\mu\mathrm{J}$.

(b) $V_4 = 6\mathrm{V}$.

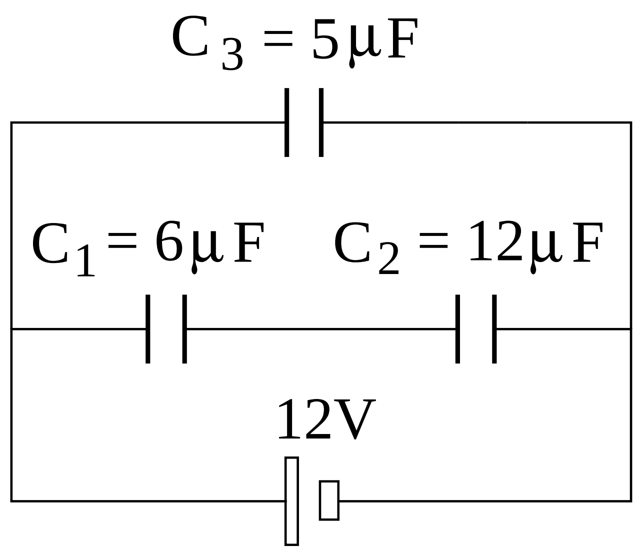
(c) $V_2 = \frac{1}{2}6\mathrm{V} = 3\mathrm{V}$.

(d) $Q_1 = (2\mu\mathrm{F})(3\mathrm{V}) = 6\mu\mathrm{C}$.

Unit Exam II: Problem #1 (Spring '08)

The circuit of capacitors is at equilibrium.

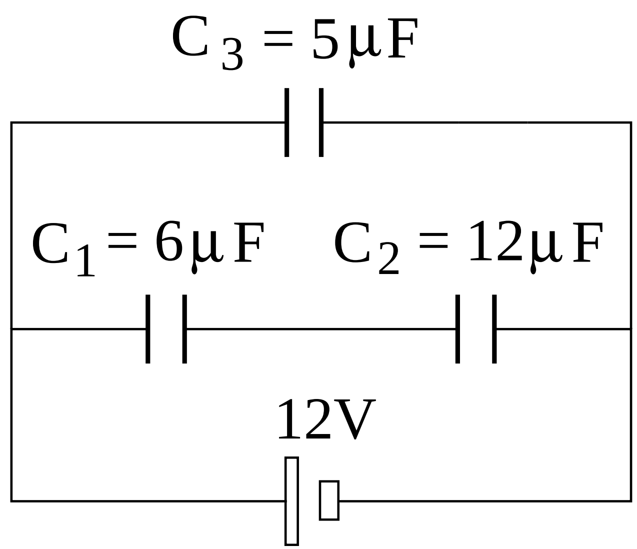
(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



Unit Exam II: Problem #1 (Spring ’08)

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.

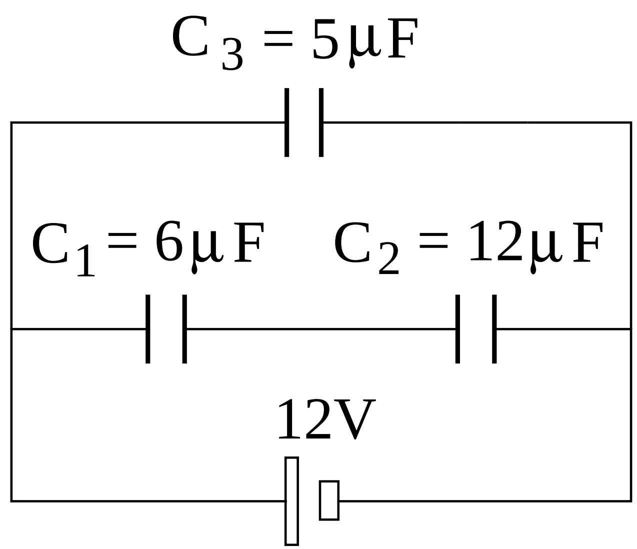


Solution:

(a) $C_{12} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}}\right)^{-1} = 4\mu\text{F},$
$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



Solution:

(a) $C_{12} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}}\right)^{-1} = 4\mu\text{F},$

$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

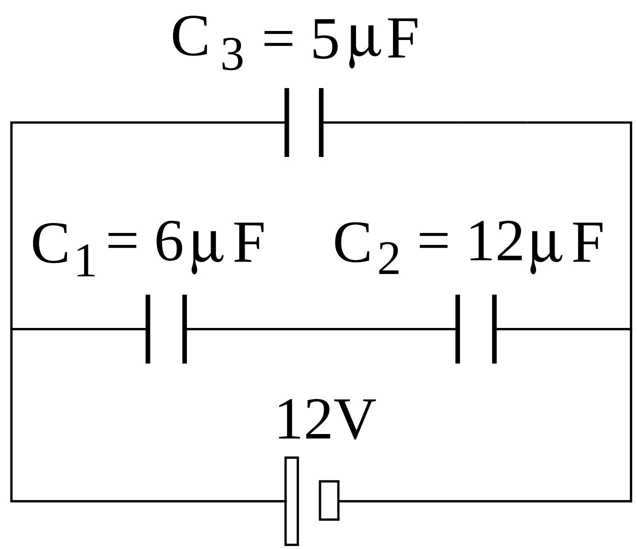
(b) $V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$

$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$

Unit Exam II: Problem #1 (Spring '08)

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



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$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

(b) $V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$

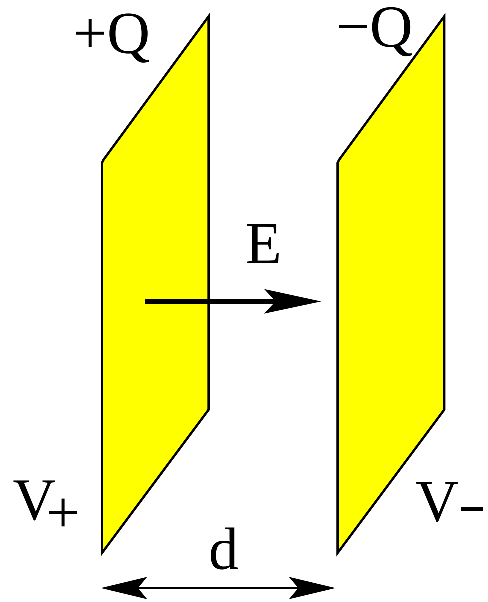
$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$

(c) $Q_3 = (5\mu\text{F})(12\text{V}) = 60\mu\text{C},$

$U_3 = \frac{1}{2}(5\mu\text{F})(12\text{V})^2 = 360\mu\text{J}.$

Unit Exam II: Problem #2 (Spring '12)

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
(b) Find the amount Q of charge on each plate.
(c) Find the energy U stored on the capacitor.
(d) Find the area A of each plate.



Unit Exam II: Problem #2 (Spring '12)

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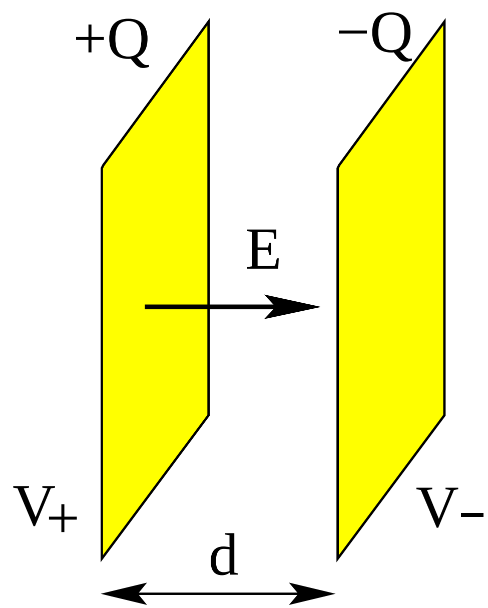

Solution:

(a) $E = \dfrac{V}{d} = \dfrac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

Unit Exam II: Problem #2 (Spring ’12)

Consider a parallel-plate capacitor of capacitance $C = 6\text{pF}$ with plates separated a distance $d = 1\text{mm}$ and a potential difference $V = V_+ - V_- = 3\text{V}$ between them.
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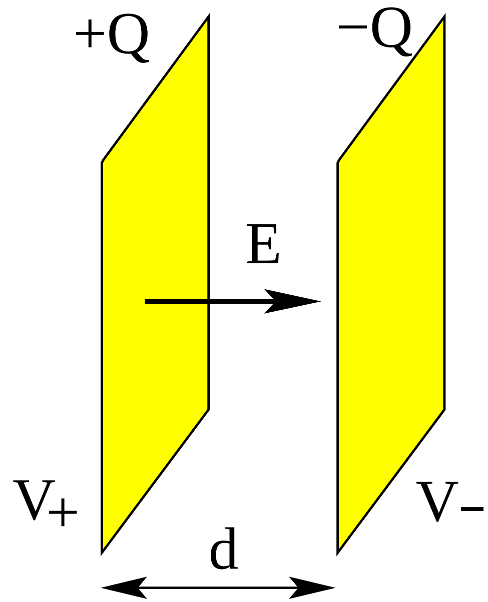

Solution:

(a) $E = \dfrac{V}{d} = \dfrac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

(b) $Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$

Unit Exam II: Problem #2 (Spring ’12)

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
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(c) Find the energy U stored on the capacitor.
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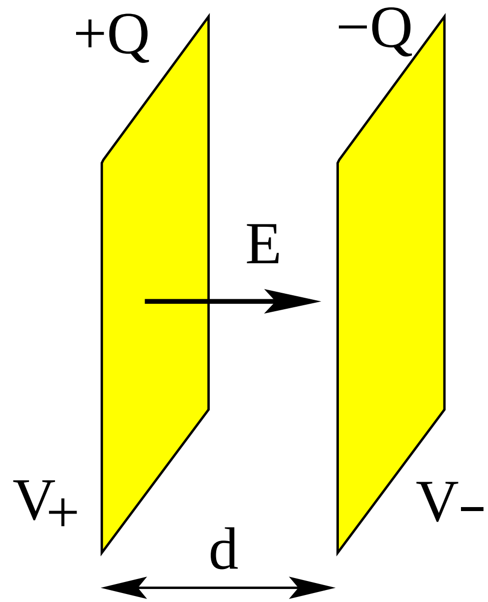


Solution:

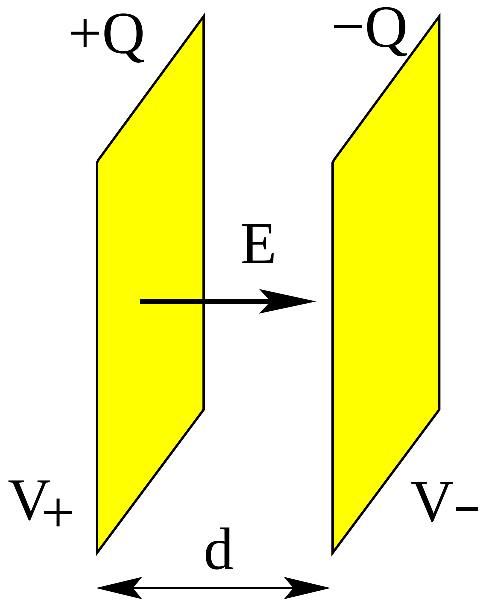
(a) $E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

(b) $Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$

(c) $U = \frac{1}{2}QV = 0.5(18\text{pC})(3\text{V}) = 27\text{pJ}.$

Unit Exam II: Problem #2 (Spring '12)

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
(b) Find the amount Q of charge on each plate.
(c) Find the energy U stored on the capacitor.
(d) Find the area A of each plate.



Solution:

(a) $E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

(b) $Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$

(c) $U = \frac{1}{2}QV = 0.5(18\text{pC})(3\text{V}) = 27\text{pJ}.$

(d) $A = \frac{Cd}{\epsilon_0} = \frac{(6\text{pF})(1\text{mm})}{8.85 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 6.78 \times 10^{-4}\text{m}^2.$