

# Magnetic Moment of a Rotating Disk



Consider a nonconducting disk of radius  $R$  with a uniform surface charge density  $\sigma$ . The disk rotates with angular velocity  $\vec{\omega}$ .

Calculation of the magnetic moment  $\vec{\mu}$ :

- Total charge on disk:  $Q = \sigma(\pi R^2)$ .
- Divide the disk into concentric rings of width  $dr$ .
- Period of rotation:  $T = \frac{2\pi}{\omega}$ .
- Current within ring:  $dI = \frac{dQ}{T} = \sigma(2\pi r dr) \frac{\omega}{2\pi} = \sigma\omega r dr$ .
- Magnetic moment of ring:  $d\mu = dI(\pi r^2) = \pi\sigma\omega r^3 dr$ .
- Magnetic moment of disk:  $\mu = \int_0^R \pi\sigma\omega r^3 dr = \frac{\pi}{4}\sigma R^4\omega$ .
- Vector relation:  $\vec{\mu} = \frac{\pi}{4}\sigma R^4\vec{\omega} = \frac{1}{4}QR^2\vec{\omega}$ .

