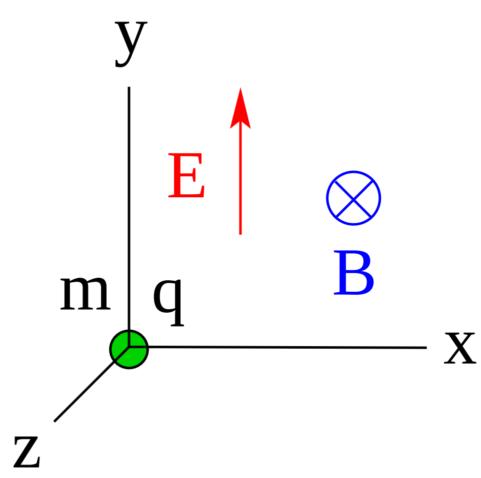
Charged Particle in Crossed Electric and Magnetic Fields (1)



- Release particle from rest.
- Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- (1) $F_x = m\frac{dv_x}{dt} = -qv_y B \qquad \Rightarrow \frac{dv_x}{dt} = -\frac{qB}{m}v_y$
- (2) $F_y = m\frac{dv_y}{dt} = qv_x B + qE \Rightarrow \frac{dv_y}{dt} = \frac{qB}{m}v_x + \frac{qE}{m}$
- Ansatz: $v_x(t) = w_x \cos(\omega_0 t) + u_x,\ v_y(t) = w_y \sin(\omega_0 t) + u_y$
- Substitute ansatz into (1) and (2) to find $w_x, w_y, u_x, u_y, \omega_0$.
- (1) $-\omega_0 w_x \sin(\omega_0 t) = -\frac{qB}{m}w_y \sin(\omega_0 t) - \frac{qB}{m}u_y$
- (2) $\omega_0 w_y \cos(\omega_0 t) = \frac{qB}{m}w_x \cos(\omega_0 t) + \frac{qB}{m}u_x + \frac{qE}{m}$
- $\Rightarrow\ u_y = 0, \quad u_x = -\frac{E}{B}, \quad \omega_0 = \frac{qB}{m}, \quad w_x = w_y \equiv w$
- Initial condition: $v_x(0) = v_y(0) = 0 \Rightarrow\ w = \frac{E}{B}$

Charged Particle in Crossed Electric and Magnetic Fields (2)

- Solution for velocity of particle:

$$v_x(t) = \frac{E}{B}\left[\cos\left(\frac{qBt}{m}\right) - 1\right], \quad v_y(t) = \frac{E}{B}\sin\left(\frac{qBt}{m}\right)$$

- Solution for position of particle:

$$x(t) = \frac{E}{B}\int_0^t \left[\cos\left(\frac{qBt}{m}\right) - 1\right] dt = \frac{Em}{qB^2}\sin\left(\frac{qBt}{m}\right) - \frac{Et}{B}$$

$$y(t) = \frac{E}{B}\int_0^t \sin\left(\frac{qBt}{m}\right) dt = \frac{Em}{qB^2}\left[1 - \cos\left(\frac{qBt}{m}\right)\right]$$

- Path of particle in (x, y)-plane: cycloid