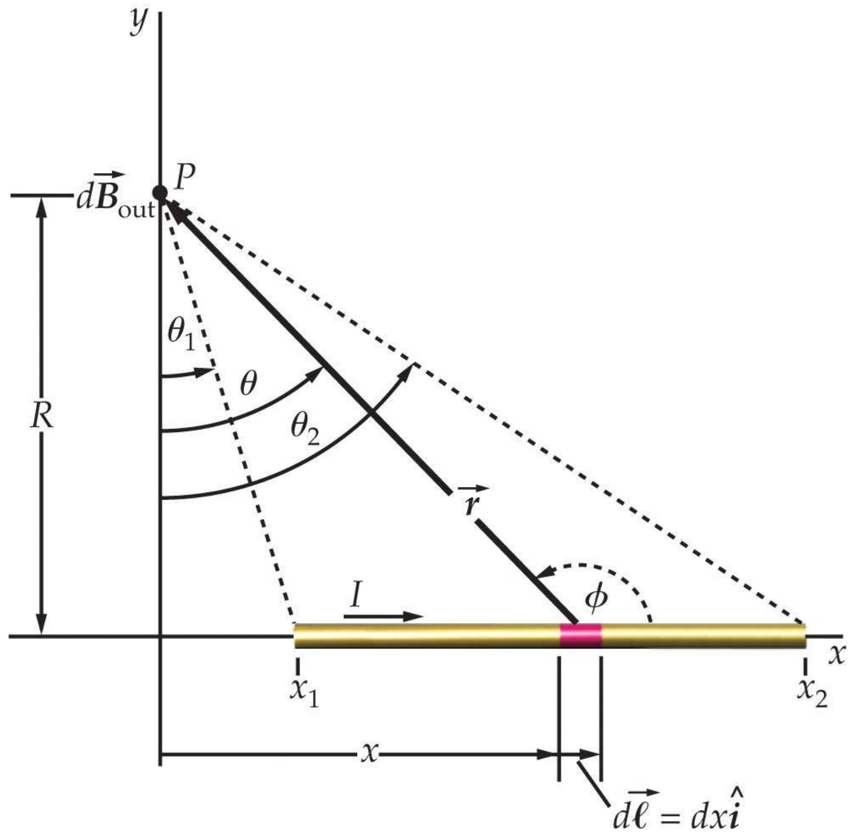
# Magnetic Field Generated by Current in Straight Wire (1)

Consider a field point $P$ that is a distance $R$ from the axi



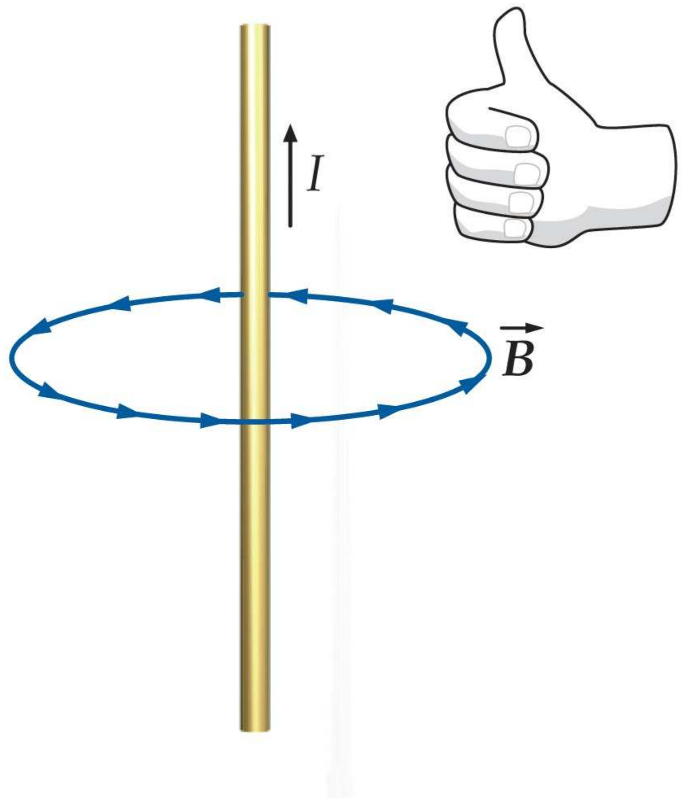

- $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idx}{r^2}\sin\phi = \dfrac{\mu_0}{4\pi}\dfrac{Idx}{r^2}\cos\theta$
- $x = R\tan\theta \;\Rightarrow\; \dfrac{dx}{d\theta} = \dfrac{R}{\cos^2\theta} = \dfrac{R}{R^2/r^2} = \dfrac{r^2}{R}$
- $dB = \dfrac{\mu_0}{4\pi}\dfrac{I}{r^2}\dfrac{r^2 d\theta}{R}\cos\theta = \dfrac{\mu_0}{4\pi}\dfrac{I}{R}\cos\theta d\theta$
- $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{R}\displaystyle\int_{\theta_1}^{\theta_2}\cos\theta d\theta$
  $= \dfrac{\mu_0}{4\pi}\dfrac{I}{R}(\sin\theta_2 - \sin\theta_1)$
- Length of wire: $L = R(\tan\theta_2 - \tan\theta_1)$

Wire of infinite length: $\theta_1 = -90^\circ,\ \theta_2 = 90^\circ \;\Rightarrow\; B = \dfrac{\mu_0 I}{2\pi R}$

# Magnetic Field Generated by Current in Straight Wire (2)

Consider a current $I$ in a straight wire of infinite length.

- The magnetic field lines are concentric circles in planes prependicular to the wire.
- The magnitude of the magnetic field at distance $R$ from the center of the wire is $B = \dfrac{\mu_0 I}{2\pi R}$.
- The magnetic field strength is proportional to the current $I$ and inversely proportional to the distance $R$ from the center of the wire.
- The magnetic field vector is tangential to the circular field lines and directed according to the right-hand rule.



# Magnetic Field Generated by Current in Straight Wire (3)

Consider the magnetic field $\vec{B}$ in the limit $R \to 0$.

- $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{R}\left(\sin\theta_2 - \sin\theta_1\right)$
- $\sin\theta_1 = \dfrac{a}{\sqrt{a^2+R^2}} = \dfrac{1}{\sqrt{1+\frac{R^2}{a^2}}} \simeq 1 - \dfrac{1}{2}\dfrac{R^2}{a^2}$
- $\sin\theta_2 = \dfrac{2a}{\sqrt{4a^2+R^2}} = \dfrac{1}{\sqrt{1+\frac{R^2}{4a^2}}} \simeq 1 - \dfrac{1}{2}\dfrac{R^2}{4a^2}$
- $B \simeq \dfrac{\mu_0}{4\pi}\dfrac{I}{R}\left(1 - \dfrac{1}{2}\dfrac{R^2}{4a^2} - 1 + \dfrac{1}{2}\dfrac{R^2}{a^2}\right)$

  $= \dfrac{\mu_0 I}{4\pi}\dfrac{3R}{8a^2} \xrightarrow{R\to 0} 0$