

# Cross Product Between Vectors



Consider two vectors  $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$  and  $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ .

- $\vec{A} \times \vec{B} = AB \sin \phi \hat{n}$ .
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ .
- $\vec{A} \times \vec{A} = 0$ .
- $\vec{A} \times \vec{B} = AB \hat{n}$  if  $\vec{A} \perp \vec{B}$ .
- $\vec{A} \times \vec{B} = 0$  if  $\vec{A} \parallel \vec{B}$ .
- $$\begin{aligned}\vec{A} \times \vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_xB_x(\hat{i} \times \hat{i}) + A_xB_y(\hat{i} \times \hat{j}) + A_xB_z(\hat{i} \times \hat{k}) \\ &\quad + A_yB_x(\hat{j} \times \hat{i}) + A_yB_y(\hat{j} \times \hat{j}) + A_yB_z(\hat{j} \times \hat{k}) \\ &\quad + A_zB_x(\hat{k} \times \hat{i}) + A_zB_y(\hat{k} \times \hat{j}) + A_zB_z(\hat{k} \times \hat{k}).\end{aligned}$$
- Use  $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ ,  
 $\hat{i} \times \hat{j} = \hat{k}$ ,  $\hat{j} \times \hat{k} = \hat{i}$ ,  $\hat{k} \times \hat{i} = \hat{j}$ .
- $\Rightarrow \vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$ .

