

RL Circuit: Current Buildup in Inductor



- Loop rule: $\mathcal{E} - IR - L \frac{dI}{dt} = 0$

- Differential equation: $L \frac{dI}{dt} = \mathcal{E} - IR \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}/R - I}{L/R}$

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \Rightarrow -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current through inductor: $I(t) = \frac{\mathcal{E}}{R} [1 - e^{-Rt/L}]$

- Rate of current change: $\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-Rt/L}$

