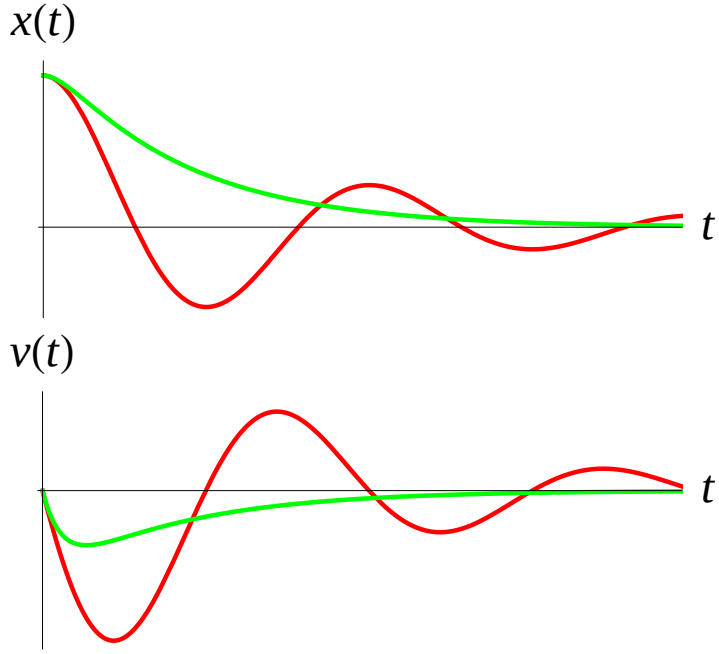
Mechanical Oscillator with Damping

- law of motion: $F = ma, \quad a = \dfrac{d^2x}{dt^2}$
- law of force: $F = -kx - bv, \quad v = \dfrac{dx}{dt}$
- equation of motion: $\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{k}{m}\,x = 0$



Solution for initial conditions $x(0) = A,\ v(0) = 0$:

(a) **underdamped motion**: $b^2 < 4km$

$$x(t) = Ae^{-bt/2m}\left[\cos(\omega' t) + \frac{b}{2m\omega'}\sin(\omega' t)\right] \quad \text{with} \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(b) **overdamped motion**: $b^2 > 4km$

$$x(t) = Ae^{-bt/2m}\left[\cosh(\Omega' t) + \frac{b}{2m\Omega'}\sinh(\Omega' t)\right] \quad \text{with} \quad \Omega' = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$