

Damped Electromagnetic Oscillator (RLC Circuit)



- loop rule: $RI + L \frac{dI}{dt} + \frac{Q}{C} = 0, I = \frac{dQ}{dt}$
- equation of motion: $\frac{d^2Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{1}{LC}Q = 0$

Solution for initial conditions $Q(0) = Q_{max}, I(0) = 0$:

(a) underdamped motion: $R^2 < \frac{4L}{C}$

$$Q(t) = Q_{max} e^{-Rt/2L} \left[\cos(\omega't) + \frac{R}{2L\omega'} \sin(\omega't) \right] \quad \text{with} \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

(b) overdamped motion: $R^2 > \frac{4L}{C}$

$$Q(t) = Q_{max} e^{-Rt/2L} \left[\cosh(\Omega't) + \frac{R}{2L\Omega'} \sinh(\Omega't) \right] \quad \text{with} \quad \Omega' = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

