

# RLC Series Circuit (1)



Applied alternating voltage:  $\mathcal{E} = \mathcal{E}_{max} \cos \omega t$

Resulting alternating current:  $I = I_{max} \cos(\omega t - \delta)$

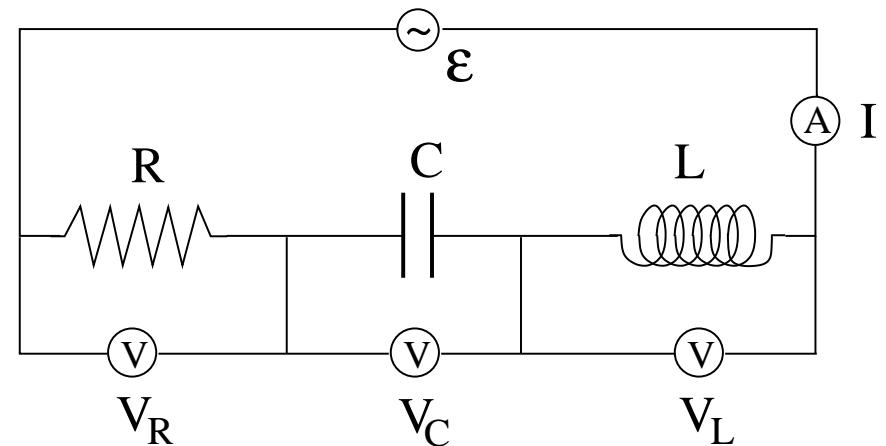
Goals:

- Find  $I_{max}, \delta$  for given  $\mathcal{E}_{max}, \omega$ .
- Find voltages  $V_R, V_L, V_C$  across devices.

Loop rule:  $\mathcal{E} - V_R - V_C - V_L = 0$

Note:

- All voltages are time-dependent.
- In general, all voltages have a different phase.
- $V_R$  has the same phase as  $I$ .



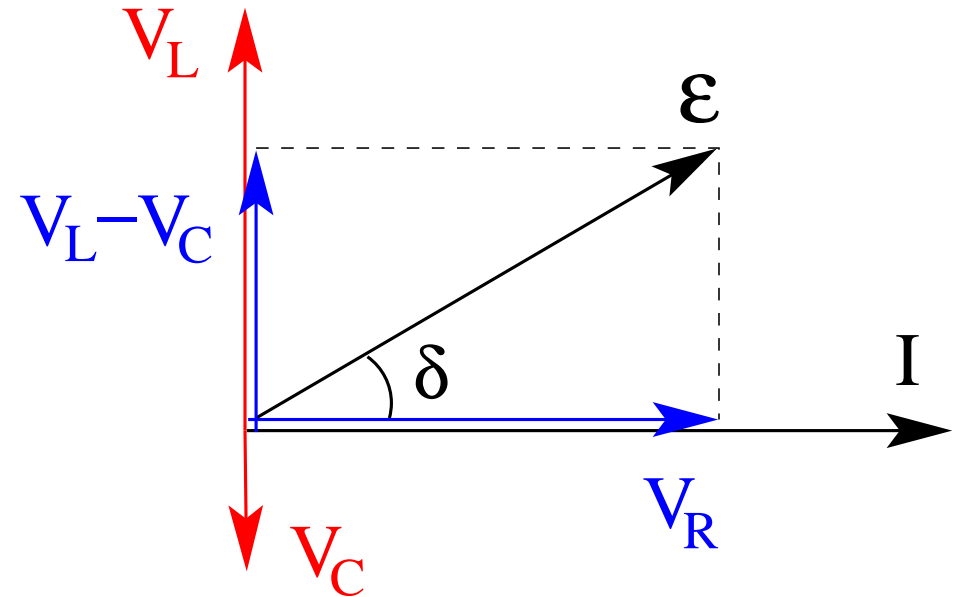
## RLC Series Circuit (2)



Phasor diagram (for  $\omega t = \delta$ ):

Voltage amplitudes:

- $V_{R,max} = I_{max}X_R = I_{max}R$
- $V_{L,max} = I_{max}X_L = I_{max}\omega L$
- $V_{C,max} = I_{max}X_C = \frac{I_{max}}{\omega C}$



Relation between  $\mathcal{E}_{max}$  and  $I_{max}$  from geometry:

$$\begin{aligned}\mathcal{E}_{max}^2 &= V_{R,max}^2 + (V_{L,max} - V_{C,max})^2 \\ &= I_{max}^2 \left[ R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2 \right]\end{aligned}$$

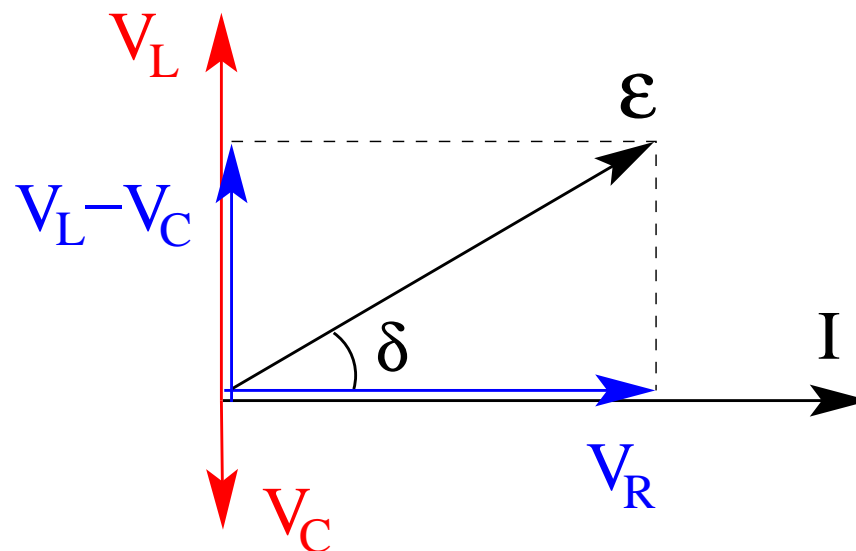
## RLC Series Circuit (3)



Impedance:  $Z \equiv \frac{\mathcal{E}_{max}}{I_{max}} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Current amplitude and phase angle:

- $I_{max} = \frac{\mathcal{E}_{max}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$
- $\tan \delta = \frac{V_{L,max} - V_{C,max}}{V_{R,max}} = \frac{\omega L - 1/\omega C}{R}$



Voltages across devices:

- $V_R = RI = RI_{max} \cos(\omega t - \delta) = V_{R,max} \cos(\omega t - \delta)$
- $V_L = L \frac{dI}{dt} = -\omega LI_{max} \sin(\omega t - \delta) = V_{L,max} \cos\left(\omega t - \delta + \frac{\pi}{2}\right)$
- $V_C = \frac{1}{C} \int I dt = \frac{I_{max}}{\omega C} \sin(\omega t - \delta) = V_{C,max} \cos\left(\omega t - \delta - \frac{\pi}{2}\right)$