RLC Parallel Circuit (1)

Applied alternating voltage: $\mathcal{E} = \mathcal{E}_{max} \cos \omega t$

Resulting alternating current: $I = I_{max} \cos(\omega t - \delta)$

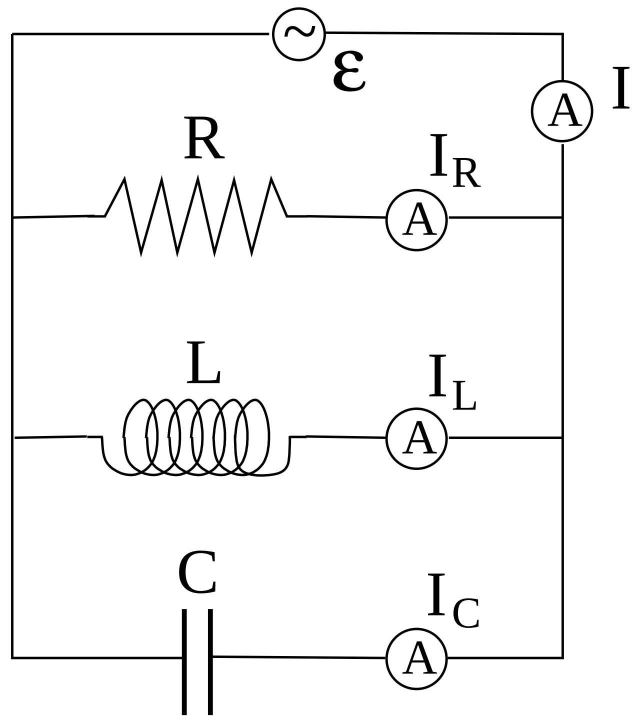
Goals:

- Find I_{max}, δ for given $\mathcal{E}_{max}, \omega$.
- Find currents I_R, I_L, I_C through devices.

Junction rule: $I = I_R + I_L + I_C$

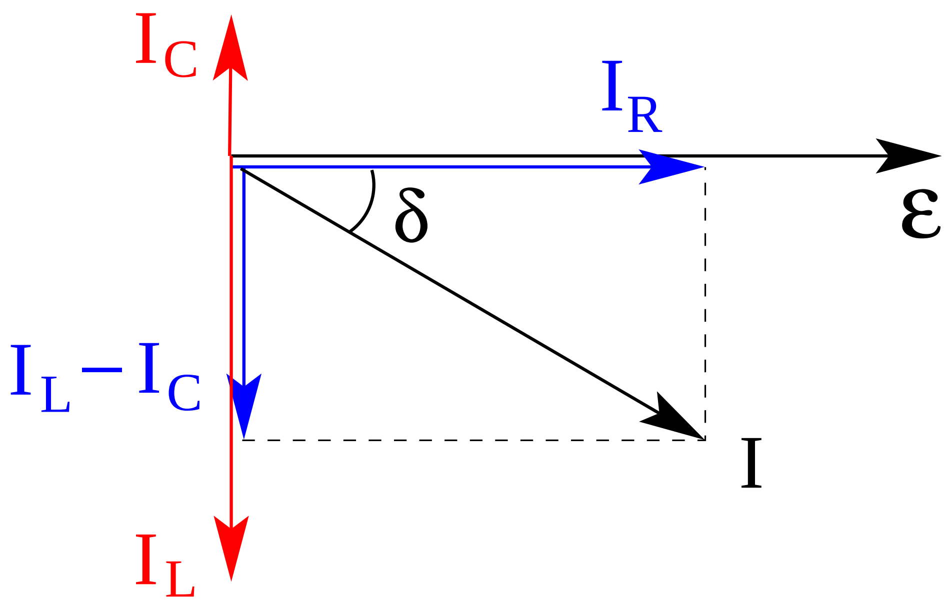
Note:

- All currents are time-dependent.
- In general, each current has a different phase
- I_R has the same phase as $\mathcal{E}$.



RLC Parallel Circuit (2)

Phasor diagram (for $\omega t = \delta$):



Current amplitudes:

- $I_{R,max} = \dfrac{\mathcal{E}_{max}}{X_R} = \dfrac{\mathcal{E}_{max}}{R}$
- $I_{L,max} = \dfrac{\mathcal{E}_{max}}{X_L} = \dfrac{\mathcal{E}_{max}}{\omega L}$
- $I_{C,max} = \dfrac{\mathcal{E}_{max}}{X_C} = \mathcal{E}_{max}\omega C$

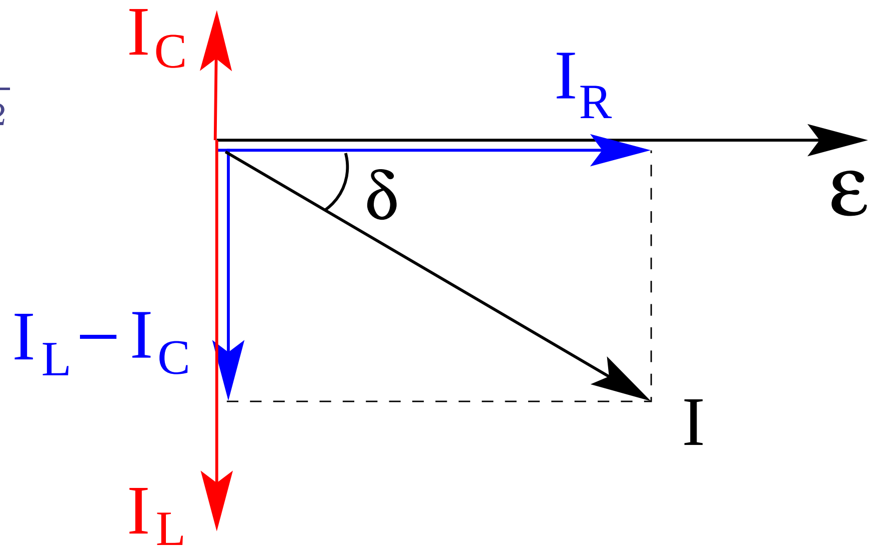
Relation between $\mathcal{E}_{max}$ and I_{max} from geometry:

$$\begin{aligned} I_{max}^2 &= I_{R,max}^2 + (I_{L,max} - I_{C,max})^2 \\ &= \mathcal{E}_{max}^2 \left[\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C \right)^2 \right] \end{aligned}$$

Impedance: $$\frac{1}{Z} \equiv \frac{I_{max}}{\mathcal{E}_{max}} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Current amplitude and phase angle:

- $$I_{max} = \frac{\mathcal{E}_{max}}{Z} = \mathcal{E}_{max}\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$
- $$\tan\delta = \frac{I_{L,max} - I_{C,max}}{I_{R,max}} = \frac{1/\omega L - \omega C}{1/R}$$



Currents through devices:

- $$I_R = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_{max}}{R}\cos(\omega t) = I_{R,max}\cos(\omega t)$$
- $$I_L = \frac{1}{L}\int \mathcal{E}dt = \frac{\mathcal{E}_{max}}{\omega L}\sin(\omega t) = I_{L,max}\cos\left(\omega t - \frac{\pi}{2}\right)$$
- $$I_C = C\frac{d\mathcal{E}}{dt} = -\omega C\mathcal{E}_{max}\sin(\omega t) = I_{C,max}\cos\left(\omega t + \frac{\pi}{2}\right)$$