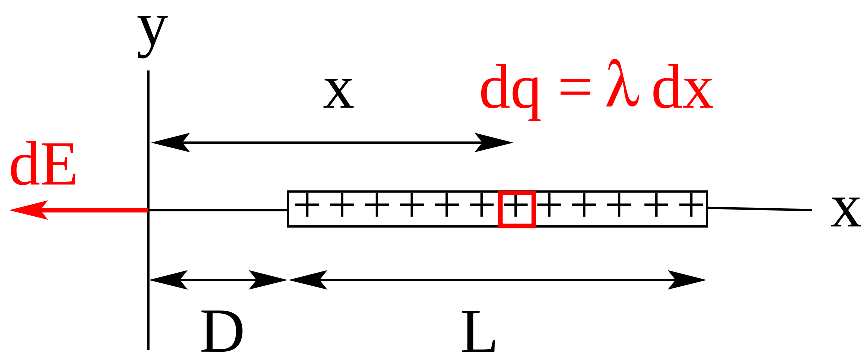
Electric Field of Charged Rod (1)

- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx: $dq = \lambda dx$



- Electric field generated by slice dx: $dE = \dfrac{kdq}{x^2} = \dfrac{k\lambda dx}{x^2}$
- Electric field generated by charged rod:

$$E = k\lambda \int_D^{D+L} \frac{dx}{x^2} = k\lambda \left[-\frac{1}{x} \right]_D^{D+L} = k\lambda \left[\frac{1}{D} - \frac{1}{D+L} \right] = \frac{kQ}{D(D+L)}$$

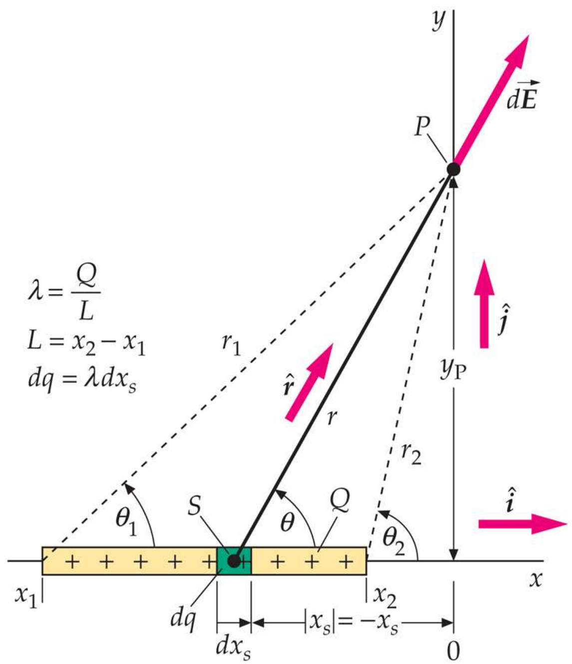
- Limiting case of very short rod $(L \ll D)$: $E \simeq \dfrac{kQ}{D^2}$
- Limiting case of very long rod $(L \gg D)$: $E \simeq \dfrac{k\lambda}{D}$

Electric Field of Charged Rod (2)

- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx_s: $dq = \lambda dx_s$
- Trigonometric relations:

$$y_p = r\sin\theta, \quad -x_s = r\cos\theta$$

$$x_s = -y_p\cot\theta, \quad dx_s = \frac{y_p d\theta}{\sin^2\theta}$$

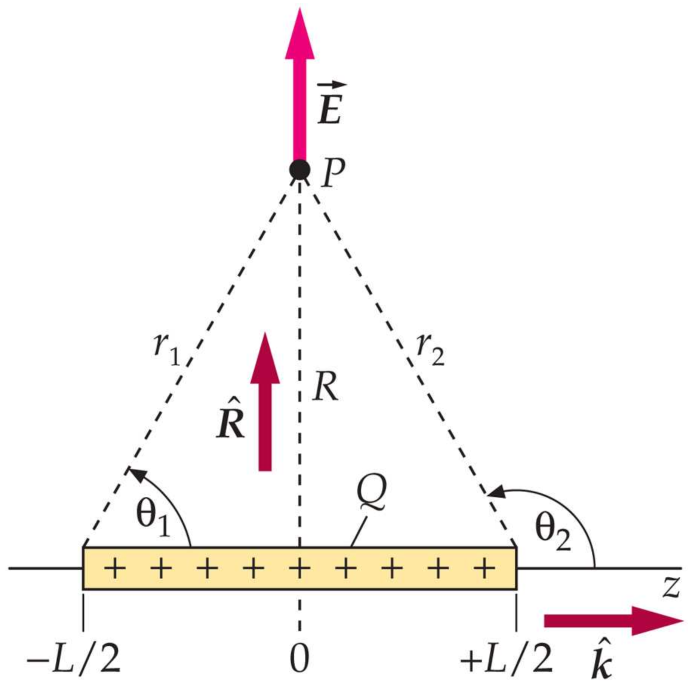


- $dE = \frac{k\lambda dx_s}{r^2} = \frac{k\lambda dx_s}{y_p^2}\sin^2\theta = \frac{k\lambda d\theta}{y_p}$
- $dE_y = dE\sin\theta = \frac{k\lambda}{y_p}\sin\theta d\theta \;\Rightarrow\; E_y = \frac{k\lambda}{y_p}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = -\frac{k\lambda}{y_p}(\cos\theta_2 - \cos\theta_1)$
- $dE_x = dE\cos\theta = \frac{k\lambda}{y_p}\cos\theta d\theta \;\Rightarrow\; E_x = \frac{k\lambda}{y_p}\int_{\theta_1}^{\theta_2}\cos\theta d\theta = \frac{k\lambda}{y_p}(\sin\theta_2 - \sin\theta_1)$

Electric Field of Charged Rod (3)

Symmetry dictates that the resulting electric field is directed radially.

- $\theta_2 = \pi - \theta_1, \quad \Rightarrow \quad \sin\theta_2 = \sin\theta_1, \quad \cos\theta_2 = -\cos\theta_1.$
- $\cos\theta_1 = \dfrac{L/2}{\sqrt{L^2/4 + R^2}}.$
- $E_R = -\dfrac{k\lambda}{R}(\cos\theta_2 - \cos\theta_1) = \dfrac{k\lambda}{R}\dfrac{L}{\sqrt{L^2/4 + R^2}}.$
- $E_z = \dfrac{k\lambda}{R}(\sin\theta_2 - \sin\theta_1) = 0.$
- Large distance $(R \gg L)$: $E_R \simeq \dfrac{kQ}{R^2}.$
- Small distances $(R \ll L)$: $E_R \simeq \dfrac{2k\lambda}{R}$
- Rod of infinite length: $\vec{E} = \dfrac{2k\lambda}{R}\hat{R}.$



Electric Field of Charged Rod (4)

Symmetry dictates that the resulting electric field is directed radially.

- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx: $dq = \lambda dx$
- $dE = \dfrac{kdq}{r^2} = \dfrac{k\lambda dx}{x^2 + y^2}$
- $dE_y = dE\cos\theta = \dfrac{dEy}{\sqrt{x^2 + y^2}} = \dfrac{k\lambda y dx}{(x^2 + y^2)^{3/2}}$
- $E_y = \displaystyle\int_{-L/2}^{+L/2} \frac{k\lambda y dx}{(x^2 + y^2)^{3/2}} = \left[\frac{k\lambda y x}{y^2\sqrt{x^2 + y^2}}\right]_{-L/2}^{+L/2}$
- $E_y = \dfrac{k\lambda L}{y\sqrt{(L/2)^2 + y^2}} = \dfrac{kQ}{y\sqrt{(L/2)^2 + y^2}}$
- Large distance $(y \gg L)$: $E_y \simeq \dfrac{kQ}{y^2}$
- Small distances $(y \ll L)$: $E_y \simeq \dfrac{2k\lambda}{y}$