Energy Transport in Electromagnetic Plane Wave

Fields: $E_y(x,t) = E_{max}\sin(kx-\omega t), \quad B_z(x,t) = B_{max}\sin(kx-\omega t)$.

Energy density: $u(x,t) = \frac{1}{2}\epsilon_0 E_y^2(x,t) + \frac{1}{2\mu_0}B_z^2(x,t). \quad$ [J/m^3]

Use the amplitude relations $\epsilon_0 E_{max}^2 = \epsilon_0 c^2 B_{max}^2 = \frac{1}{\mu_0}B_{max}^2$.

$$u(x,t) = \epsilon_0 E_{max}^2 \sin^2(kx-\omega t) = \frac{1}{\mu_0}B_{max}^2\sin^2(kx-\omega t) = \frac{E_{max}B_{max}}{c\mu_0}\sin^2(kx-\omega t).$$

Energy transported across area A in time dt: $dU(x,t) = u(x,t)Acdt. \quad$ [J]

Power transported per unit area: $\frac{1}{A}\frac{dU}{dt} = u(x,t)c = S(x,t). \quad$ [W/m^2]

Intensity (average power transported per unit area):

$$I = \bar{S} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{\epsilon_0 c}{2}E_{max}^2 = \frac{c}{2\mu_0}B_{max}^2. \quad \text{[W/m}^2\text{]}$$