Electric Field of Charged Semicircle

Consider a uniformly charged thin rod bent into a semicircle of radius R.

Find the electric field generated at the origin of the coordinate system.

- Charge per unit length: $\lambda = Q/\pi R$
- Charge on slice: $dq = \lambda R d\theta$ (assumed positive)
- Electric field generated by slice: $dE = k\dfrac{|dq|}{R^2} = \dfrac{k|\lambda|}{R}d\theta$ directed radially (inward for $\lambda > 0$)
- Components of $d\vec{E}$: $dE_x = dE\cos\theta, \quad dE_y = -dE\sin\theta$
- Electric field from all slices added up:

$$E_x = \frac{k\lambda}{R}\int_0^\pi \cos\theta\, d\theta = \frac{k\lambda}{R}\left[\sin\theta\right]_0^\pi = 0$$

$$E_y = -\frac{k\lambda}{R}\int_0^\pi \sin\theta\, d\theta = \frac{k\lambda}{R}\left[\cos\theta\right]_0^\pi = -\frac{2k\lambda}{R}$$