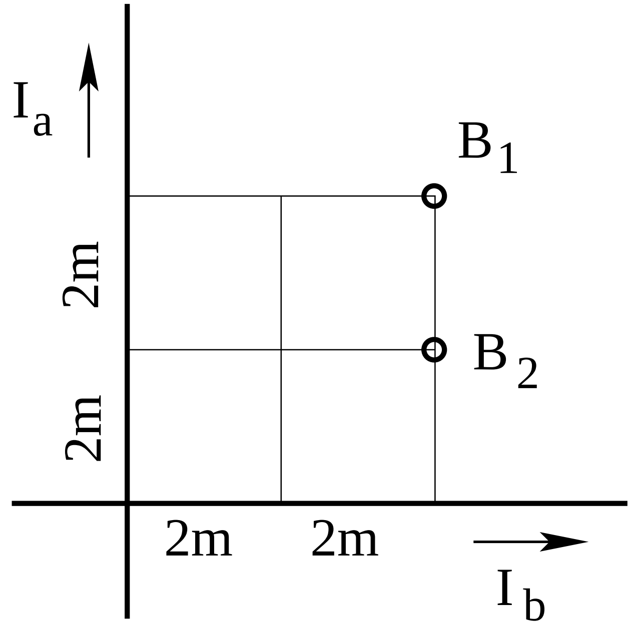
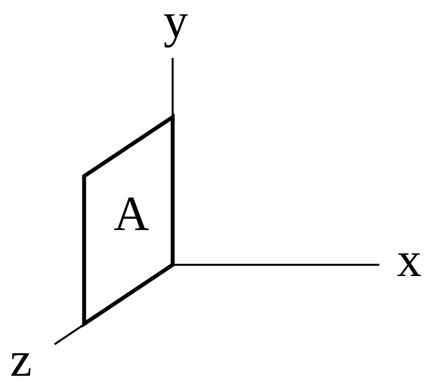
Intermediate Exam III: Problem #1 (Spring '06)

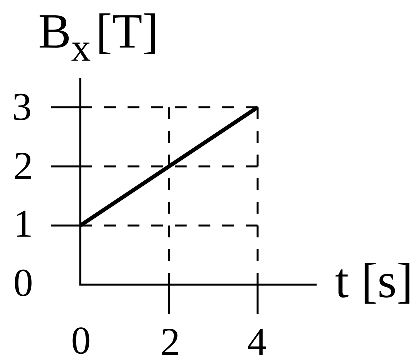
Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5$A in the directions shown.
Find the direction (in/out) and the magnitude of the magnetic fields $\mathbf{B}_1$ and $\mathbf{B}_2$ at the points marked in the graph.



- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{4\text{m}} - \frac{5\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{2\text{m}} - \frac{5\text{A}}{4\text{m}}\right) = 0.25\mu\text{T}$ (out of plane).

A conducting loop in the shape of a square with area $A = 4\mathrm{m}^2$ and resistance $R = 5\Omega$ is placed in the yz-plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.
(a) Find the magnetic flux Φ_B through the loop at time $t = 0$.
(b) Find magnitude and direction (cw/ccw) of the induced current I at time $t = 2$s.





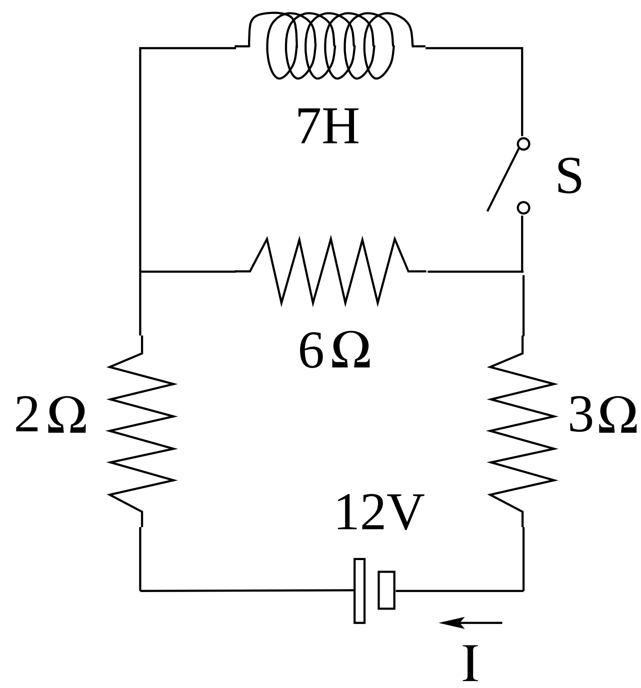
Choice of area vector: $\odot/\otimes$ $\Rightarrow$ positive direction = ccw/cw.

(a) $\Phi_B = \pm(1\mathrm{T})(4\mathrm{m}^2) = \pm 4\mathrm{Tm}^2.$

(b) $\dfrac{d\Phi_B}{dt} = \pm(0.5\mathrm{T/s})(4\mathrm{m}^2) = \pm 2\mathrm{V} \quad \Rightarrow \quad \mathcal{E} = -\dfrac{d\Phi_B}{dt} = \mp 2\mathrm{V}.$

$\Rightarrow \; I = \dfrac{\mathcal{E}}{R} = \mp\dfrac{2\mathrm{V}}{5\Omega} = \mp 0.4\mathrm{A}$ (cw).

In the circuit shown the switch S is initially open.
Find the current I through the battery
(a) while the switch is open,
(b) immediately after the switch has been closed,
(c) a very long time later.



(a) $I = \dfrac{12\mathrm{V}}{2\Omega + 3\Omega + 6\Omega} = 1.09\mathrm{A}.$

(b) $I = \dfrac{12\mathrm{V}}{2\Omega + 3\Omega + 6\Omega} = 1.09\mathrm{A}.$

(c) $I = \dfrac{12\mathrm{V}}{2\Omega + 3\Omega} = 2.4\mathrm{A}.$

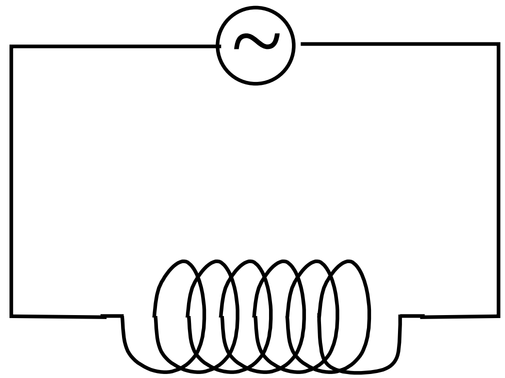
Intermediate Exam III: Problem #4 (Spring ’06)

Consider the circuit shown. The ac voltage supplied is $\mathcal{E} = \mathcal{E}_{max}\cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value I_{max} of the current?

(b) What is the emf $\mathcal{E}$ at $t = 0.02$s?

(c) What is the current I at $t = 0.02$s?



(a) $I_{max} = \frac{\mathcal{E}_{max}}{X_L} = \frac{\mathcal{E}_{max}}{\omega L} = \frac{170\mathrm{V}}{11.3\Omega} = 15.0\mathrm{A}.$

(b) $\mathcal{E} = \mathcal{E}_{max}\cos(7.54\mathrm{rad}) = (170\mathrm{V})(0.309) = 52.5\mathrm{V}.$

(c) $I = I_{max}\cos(7.54\mathrm{rad} - \pi/2) = (15.0\mathrm{A})(0.951) = 14.3\mathrm{A}.$