Electric Flux: Definition

Consider a surface S of arbitrary shape in the presence of an electric field $\vec{E}$.
Prescription for the calculation of the electric flux through S:

- Divide S into small tiles of area ΔA_i.
- Introduce vector $\Delta\vec{A}_i = \hat{n}_i \Delta A_i$ perpendicular to tile.
  - If S is open choose consistently one of two possible directions for $\Delta\vec{A}_i$.
  - If S is closed choose $\Delta\vec{A}_i$ to be directed outward.
- Electric field at position of tile i: $\vec{E}_i$.
- Electric flux through tile i:
  $\Delta\Phi_i^{(E)} = \vec{E}_i \cdot \Delta\vec{A}_i = E_i \Delta A_i \cos\theta_i$.
- Electric flux through S: $\Phi_E = \sum_i \vec{E}_i \cdot \Delta\vec{A}_i$.
- Limit of infinitesimal tiles: $\Phi_E = \int \vec{E} \cdot d\vec{A}$.
- Electric flux is a scalar.
- The SI unit of electric flux is Nm^2/C.