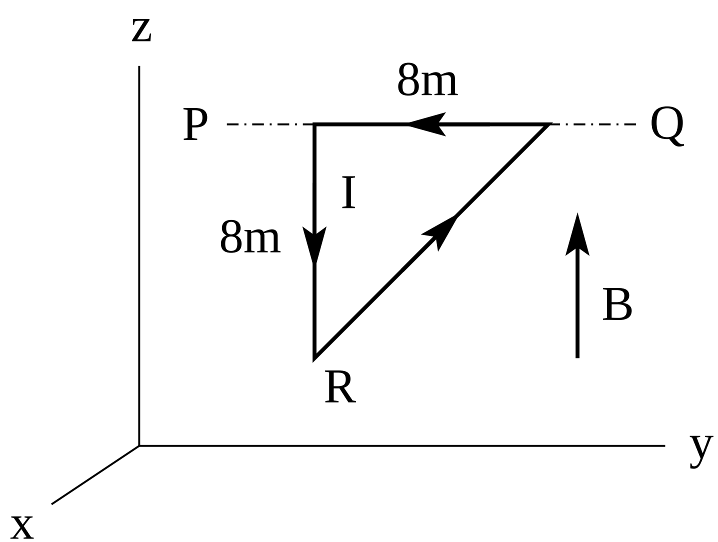
Unit Exam III: Problem #1 (Spring ’09)

A triangular conducting loop in the yz-plane with a counterclockwise current $I = 3$A is free to rotate about the axis PQ. A uniform magnetic field $\vec{B} = 0.5\mathrm{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle.
(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle.
(c) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



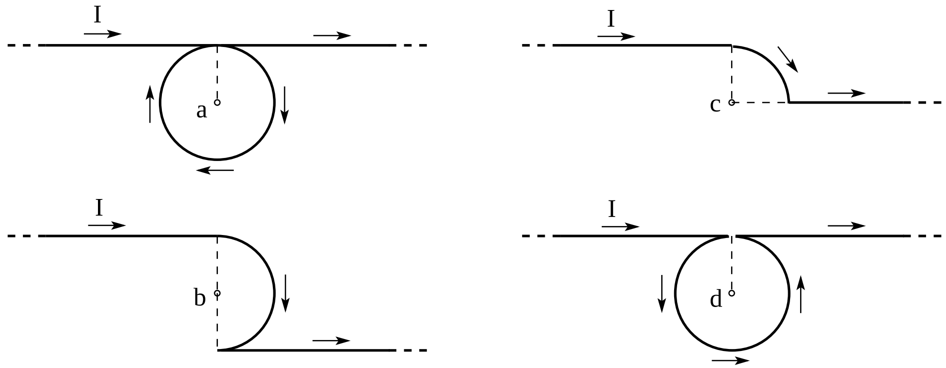
Solution:

(a) $\vec{\mu} = (3\mathrm{A})(32\mathrm{m}^2)\hat{i} = 96\mathrm{Am}^2\hat{i}$.

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\mathrm{Am}^2\hat{i}) \times (0.5\mathrm{T}\hat{k}) = -48\mathrm{Nm}\hat{j}$.

(c) $(-8\mathrm{m}\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48\mathrm{Nm}\hat{j} \quad \Rightarrow \quad \vec{F}_R = -6\mathrm{N}\hat{i}$.

Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1$m in four different configurations. A current $I = 1$A flows in the directions shown. Find magnitude B_a, B_b, B_c, B_d and direction ($\odot/\otimes$) of the magnetic field thus generated at the points a, b, c, d.



Solution:

$$B_a = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} + 628\text{nT} + 100\text{nT}| = 828\text{nT} \quad \otimes$$

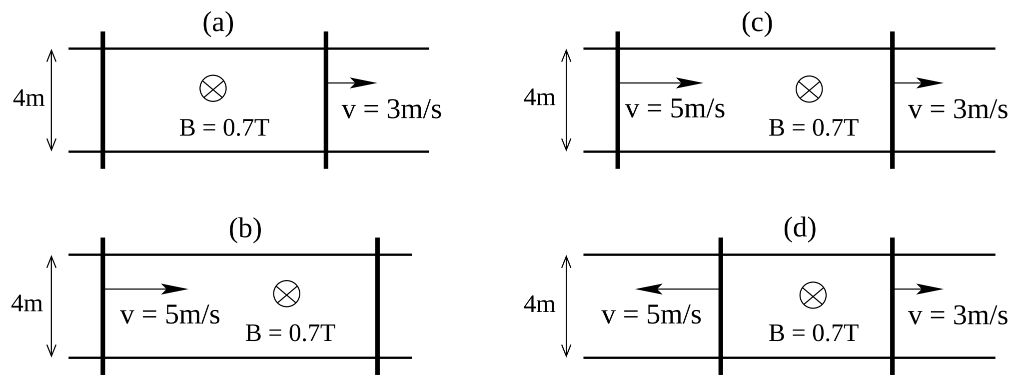
$$B_b = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} + 314\text{nT} - 100\text{nT}| = 314\text{nT} \quad \otimes$$

$$B_c = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{8R} + 0\right| = |100\text{nT} + 157\text{nT}| = 257\text{nT} \quad \otimes$$

$$B_d = \left|\frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} - 628\text{nT} + 100\text{nT}| = 428\text{nT} \quad \odot$$

Unit Exam III: Problem #3 (Spring '09)

A pair of rails are connected by two mobile rods. A uniform magnetic field B directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude I and direction (cw/ccw) of the induced current in each case.



Solution:

(a) $|\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A}$ ccw

(b) $|\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, \quad I = \frac{14\text{V}}{0.2\Omega} = 70\text{A}$ cw

(c) $|\mathcal{E}| = (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, \quad I = \frac{5.6\text{V}}{0.2\Omega} = 28\text{A}$ cw

(d) $|\mathcal{E}| = (5\text{m/s} + 3\text{m/s})(0.7\text{T})(4\text{m}) = 22.4\text{V}, \quad I = \frac{22.4\text{V}}{0.2\Omega} = 112\text{A}$ ccw