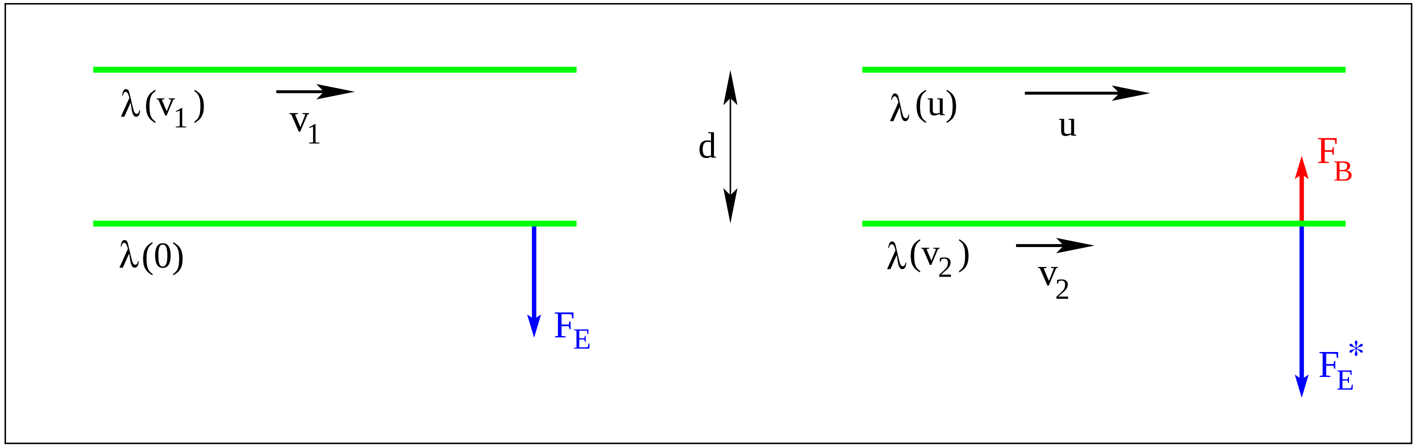
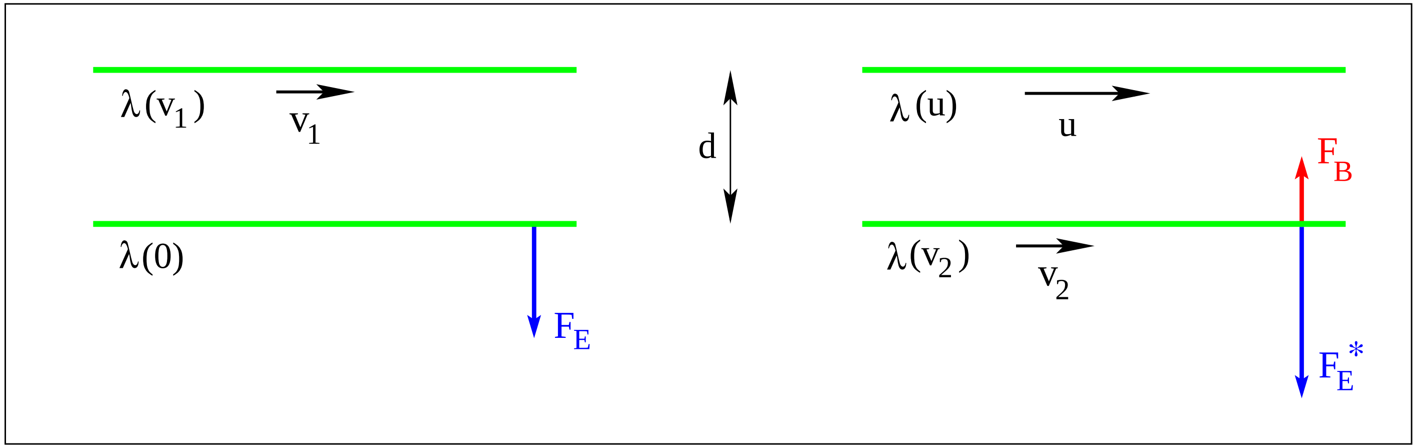
Catching Up with a Photon? (1)

Forces between two long, parallel, charged rods in relative motion.



- Galilean kinematics predicts $u = v_1 + v_2$.
- Relativistic kinematics requires $v_1 < c,\ v_2 < c,\ u < c$.
- Relativistic dynamics requires $F_E^* - F_B = F_E$.
- Length-contracted charge densities: $\lambda(v) = \dfrac{\lambda(0)}{\sqrt{1 - v^2/c^2}}$.
- Electric currents: $I(v) = \lambda(v)v$.

Catching Up with a Photon? (2)



- $\dfrac{F_E}{L} = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda(0)\lambda(v_1)}{d}, \qquad \dfrac{F_E^*}{L} = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda(v_2)\lambda(u)}{d}.$
- $\dfrac{F_B}{L} = \dfrac{\mu_0}{2\pi}\dfrac{[\lambda(v_2)v_2][\lambda(u)u]}{d} = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda(v_2)\lambda(u)}{d}\dfrac{v_2 u}{c^2}.$
- $\dfrac{F_E^* - F_B}{L} = \dfrac{F_E}{L} \quad \Rightarrow \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda(v_2)\lambda(u)}{d}\left(1 - \dfrac{v_2 u}{c^2}\right) = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda(0)\lambda(v_1)}{d}$
- $\Rightarrow \dfrac{1}{\sqrt{1 - v_2^2 c^2}}\dfrac{1}{\sqrt{1 - u^2/c^2}}\left(1 - \dfrac{v_2 u}{c^2}\right) = \dfrac{1}{\sqrt{1 - v_1^2/c^2}}$ to be solved for u.
- Relativistic kinematic predicts $u = \dfrac{v_1 + v_2}{1 + v_1 v_2/c^2} < c$.