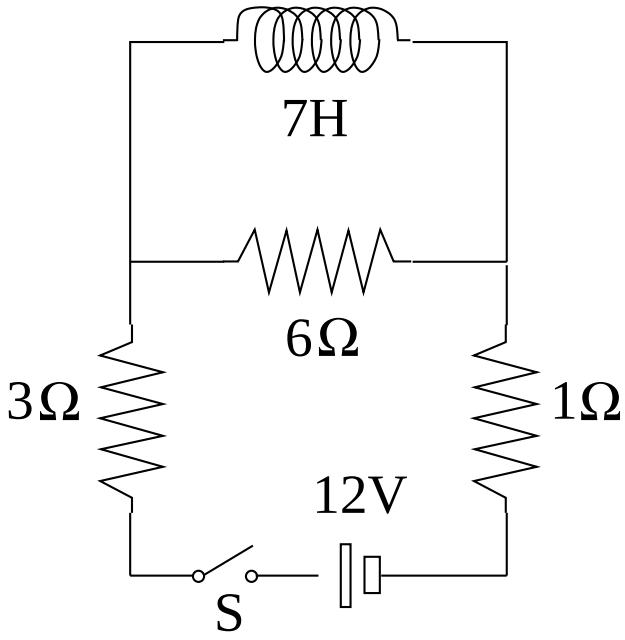
Unit Exam IV: Problem #1 (Spring '12)

In the circuit shown we close the switch S at time $t = 0$. Find the current I_L through the inductor and the voltage V_6 across the 6Ω-resistor
(a) immediately after the switch has been closed,
(b) a very long time later.



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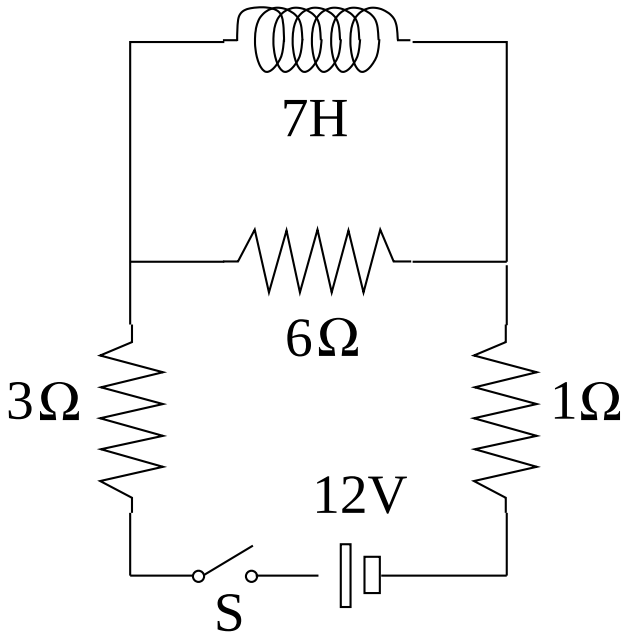


Solution:

(a) $I_L = 0, \quad I_6 = \dfrac{12\text{V}}{10\Omega} = 1.2\text{A}, \quad V_6 = (6\Omega)(1.2\text{A}) = 7.2\text{V}.$

(b) $I_L = \dfrac{12\text{V}}{4\Omega} = 3\text{A}, \quad V_6 = 0.$

Unit Exam IV: Problem #2 (Spring '12)

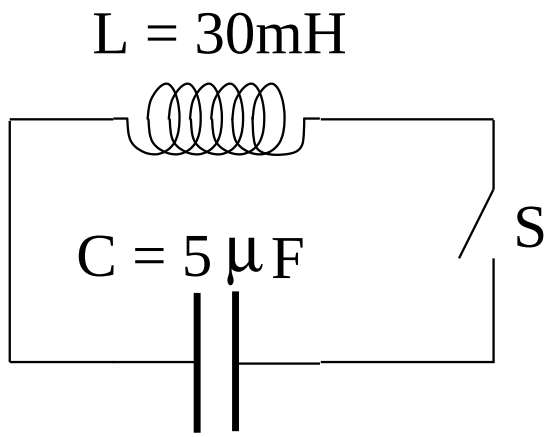
At time $t = 0$ the capacitor is charged to $Q_{max} = 4\mu$C and the switch is being closed. The charge on the capacitor begins to decrease and the current through the inductor begins to increase.

(a) At what time t_1 is the capacitor discharged for the first time?

(b) At what time t_2 has the current through the inductor returned to zero for the first time?

(c) What is the maximum energy stored in the capacitor at any time?

(d) What is the maximum energy stored in the inductor at any time?



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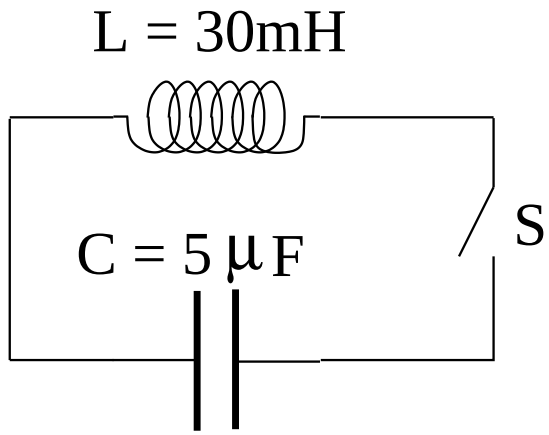


Solution:

(a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2.43\text{ms}, \qquad t_1 = \dfrac{T}{4} = 0.608\text{ms}.$

(b) $t_2 = \dfrac{T}{2} = 1.22\text{ms}.$

(c) $U_C^{max} = \dfrac{Q_{max}^2}{2C} = 1.6\mu\text{J}.$

(d) $U_L^{max} = U_C^{max} = 1.6\mu\text{J}$ (energy conservation.)

Unit Exam IV: Problem #3 (Spring '12)

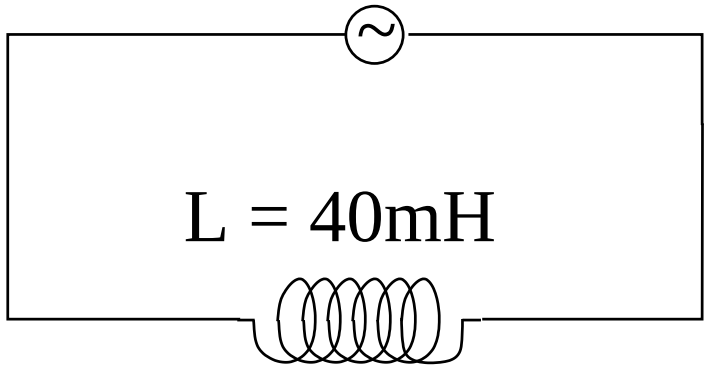
The ac voltage supplied in the circuit shown is $\mathcal{E} = \mathcal{E}_{max}\cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value I_{max} of the current?

(b) What is the emf $\mathcal{E}(t)$ at $t = 5$ms?

(c) What is the current $I(t)$ at $t = 5$ms?

(d) What is the power transfer $P(t)$ between ac source and device at $t = 5$ms?



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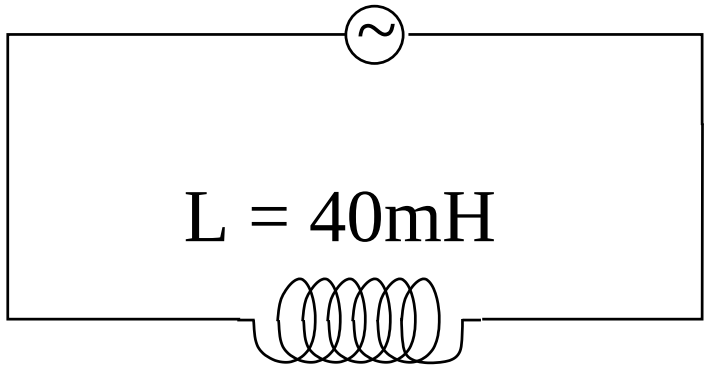

Solution:

(a) $I_{max} = \dfrac{\mathcal{E}_{max}}{\omega L} = \dfrac{170\text{V}}{(377\text{rad/s})(40\text{mH})} = 11.3\text{A}.$

(b) $\mathcal{E} = (170\text{V})\cos(1.885\text{rad}) = (170\text{V})(-0.309) = -52.5\text{V}.$

(c) $I = (11.3\text{A})\cos(1.885\text{rad} - \pi/2) = (11.3\text{A})\cos(0.314) = (11.3\text{A})(0.951) = 10.7\text{A}.$

(d) $P = \mathcal{E}I = (-52.5\text{V})(10.7\text{A}) = -562\text{W}.$