Unit Exam I: Problem #1 (Spring '13)

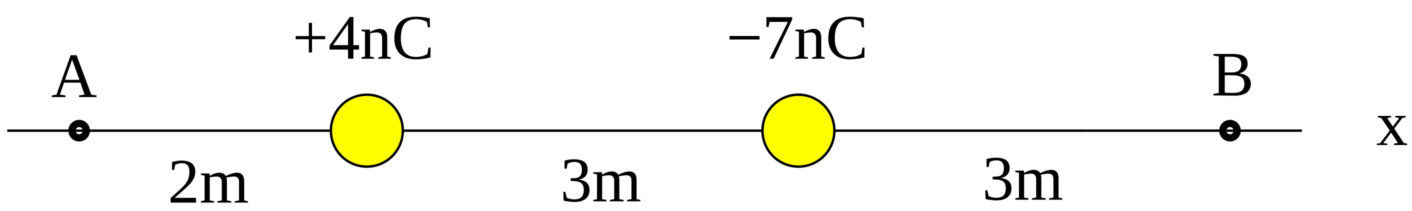
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at points A and B.

(b) Find the electric potential at points A and B.

(c) Find the electric potential energy of a proton (mass $m = 1.67 \times 10^{-27}$kg, charge $q = 1.60 \times 10^{-19}$C) when placed at point A or point B.

(d) Find magnitude and direction of the acceleration the proton experiences when released at point A or point B.



Unit Exam I: Problem #1 (Spring '13)

Solution:

(a) $$E_x = -k\frac{4\text{nC}}{(2\text{m})^2} - k\frac{(-7\text{nC})}{(5\text{m})^2} = -9.00\text{N/C} + 2.52\text{N/C} = -6.48\text{N/C}.$$

$$E_x = k\frac{4\text{nC}}{(6\text{m})^2} + k\frac{(-7\text{nC})}{(3\text{m})^2} = 1.00\text{N/C} - 7.00\text{N/C} = -6.00\text{N/C}.$$

(b) $$V = +k\frac{4\text{nC}}{2\text{m}} + k\frac{(-7\text{nC})}{5\text{m}} = 18.0\text{V} - 12.6\text{V} = 5.4\text{V}.$$

$$V = +k\frac{4\text{nC}}{6\text{m}} + k\frac{(-7\text{nC})}{3\text{m}} = 6.0\text{V} - 21.0\text{V} = -15.0\text{V}.$$

(c) $$U = qV = (5.4\text{V})(1.6\times 10^{-19}\text{C}) = 8.64\times 10^{-19}\text{J}.$$

$$U = qV = (-15.0\text{V})(1.6\times 10^{-19}\text{C}) = -2.40\times 10^{-18}\text{J}.$$

(d) $$a_x = \frac{qE_x}{m} = \frac{(1.6\times 10^{-19}\text{C})(-6.48\text{N/C})}{1.67\times 10^{-27}\text{kg}} = -6.21\times 10^{8}\text{ms}^{-2}.$$

$$a_x = \frac{qE_x}{m} = \frac{(1.6\times 10^{-19}\text{C})(-6.00\text{N/C})}{1.67\times 10^{-27}\text{kg}} = -5.75\times 10^{8}\text{ms}^{-2}.$$

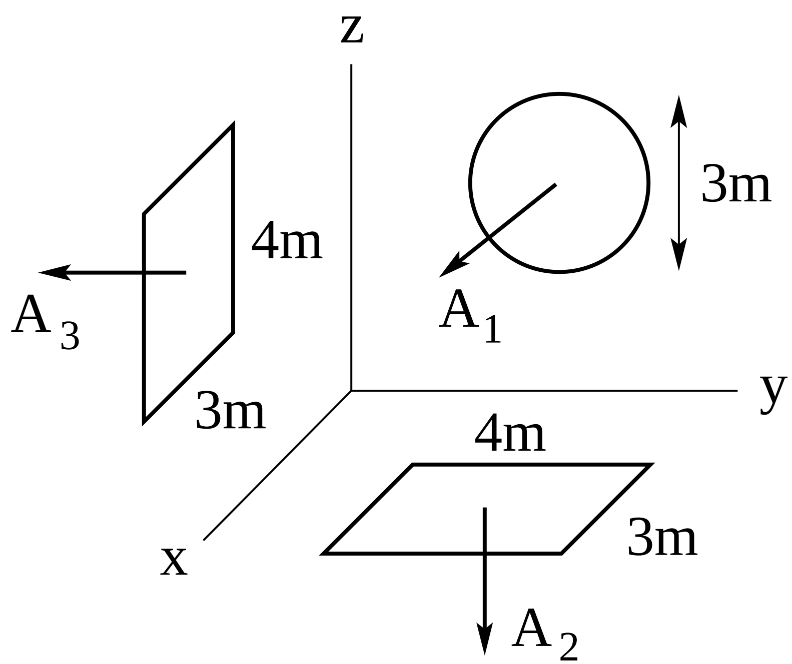
Unit Exam I: Problem #2 (Spring '13)

Consider three plane surfaces (one circle and two rectangles) with area vectors $\vec{A}_1$ (pointing in positive x-direction), $\vec{A}_2$ (pointing in negative z-direction), and $\vec{A}_3$ (pointing in negative y-direction) as shown. The region is filled with a uniform electric field $\vec{E} = (-3\hat{i} + 9\hat{j} - 4\hat{k})$N/C or $\vec{E} = (2\hat{i} - 6\hat{j} + 5\hat{k})$N/C.

(a) Find the electric flux $\Phi_E^{(1)}$ through surface 1.

(b) Find the electric flux $\Phi_E^{(2)}$ through surface 2.

(c) Find the electric flux $\Phi_E^{(3)}$ through surface 3.



Solution:

(a) $\vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (-3\text{N/C})(7.07\text{m}^2) = -21.2\text{Nm}^2/\text{C}.$

$\vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (2\text{N/C})(7.07\text{m}^2) = 14.1\text{Nm}^2/\text{C}.$

(b) $\vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (-4\text{N/C})(-12\text{m}^2) = 48\text{Nm}^2/\text{C}.$

$\vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (5\text{N/C})(-12\text{m}^2) = -60\text{Nm}^2/\text{C}.$

(b) $\vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E}\cdot\vec{A}_3 = (9\text{N/C})(-12\text{m}^2) = -108\text{Nm}^2/\text{C}.$

$\vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E}\cdot\vec{A}_3 = (-6\text{N/C})(-12\text{m}^2) = 72\text{Nm}^2/\text{C}.$

Unit Exam I: Problem #3 (Spring '13)

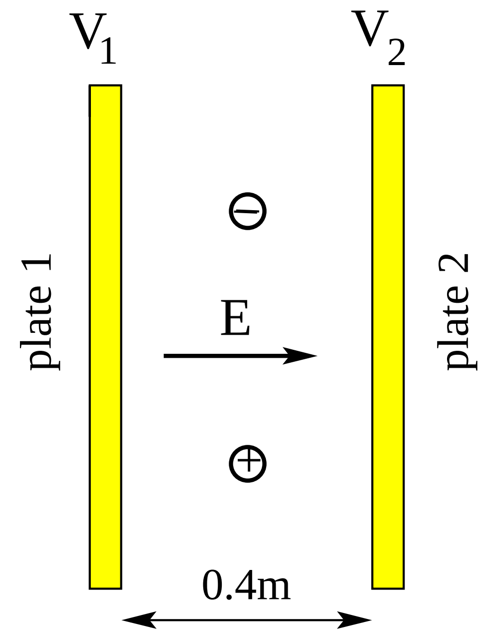
An electron ($m_e = 9.11 \times 10^{-31}$kg, $q_e = -1.60 \times 10^{-19}$C) and a proton ($m_p = 1.67 \times 10^{-27}$kg, $q_p = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The electric field between the plates is uniform and has strength $E = 40$V/m. Ignore gravity.

(a) Which plate is positively (negatively) charged?

(b) Find the electric forces $\vec{F}_p$ acting on the proton and $\vec{F}_e$ acting on the electron (magnitude and direction).

(c) Find the accelerations $\vec{a}_p$ of the proton and $\vec{a}_e$ of the electron (magnitude and direction).

(d) If plate 1 is at potential $V_1 = 1$V at what potential V_2 is plate 2?
If plate 2 is at potential $V_2 = 2$V at what potential V_1 is plate 1?



Solution:

(a) plate 1 (plate 2)

(b) $F_p = |q_p|E = 6.40 \times 10^{-18}\mathrm{N}.$ (directed right).

$F_e = |q_e|E = 6.40 \times 10^{-18}\mathrm{N}.$ (directed left).

(c) $a_p = F_p/m_p = 3.83 \times 10^{9}\mathrm{m/s^2}.$ (directed right).

$a_e = F_e/m_e = 7.03 \times 10^{12}\mathrm{m/s^2}.$ (directed left).

(d) $V_2 = 1\mathrm{V} - (40\mathrm{V/m})(0.4\mathrm{m}) = -15\mathrm{V}.$

$V_1 = 2\mathrm{V} + (40\mathrm{V/m})(0.4\mathrm{m}) = 18\mathrm{V}.$