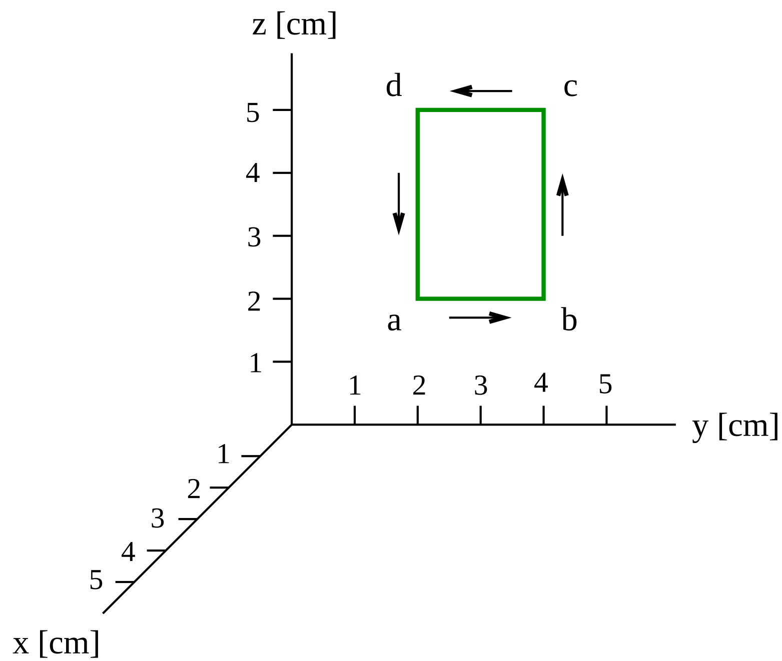
Unit Exam III: Problem #1 (Spring '14)

A counterclockwise current $I = 1.7$A [$I = 1.3$A] is flowing through the conducting rectangular frame shown in a region of magnetic field $\mathbf{B} = 6$mT$\hat{\mathbf{j}}$ [$\mathbf{B} = 6$mT$\hat{\mathbf{k}}$].
(a) Find the force $\mathbf{F}_{bc}$ [$\mathbf{F}_{ab}$] (magnitude and direction) acting on side bc [ab] of the rectangle.
(b) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



Unit Exam III: Problem #1 (Spring '14)

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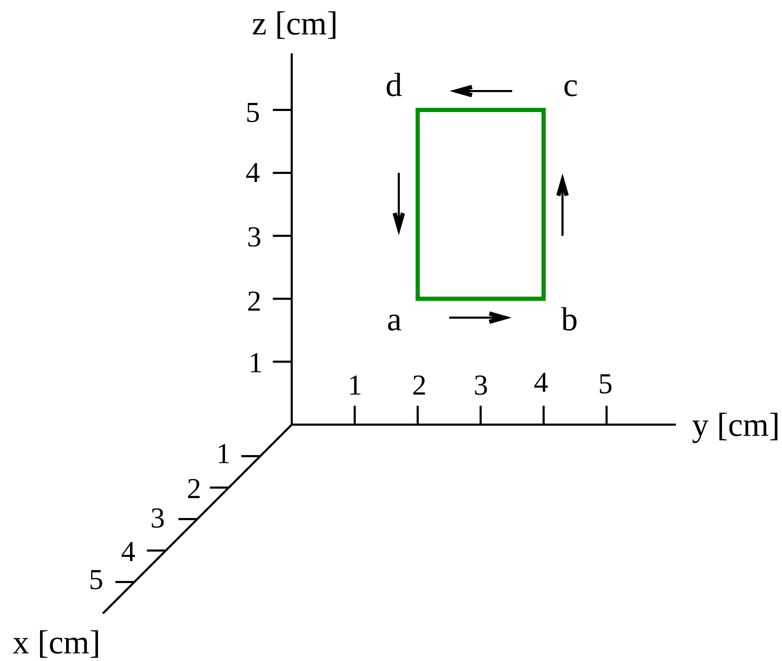


Solution:

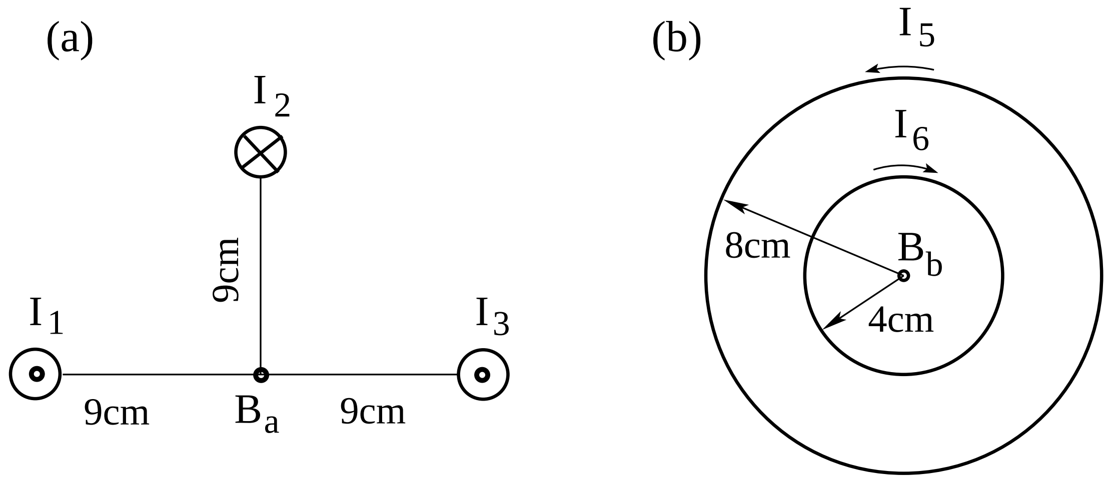
(a) $\mathbf{F}_{bc} = (1.7\text{A})(3\text{cm}\hat{\mathbf{k}}) \times (6\text{mT}\hat{\mathbf{j}}) = -3.06 \times 10^{-4}\text{N}\hat{\mathbf{i}}$
$[\mathbf{F}_{ab} = (1.3\text{A})(2\text{cm}\hat{\mathbf{j}}) \times (6\text{mT}\hat{\mathbf{k}}) = 1.56 \times 10^{-4}\text{N}\hat{\mathbf{i}}]$

(b) $\vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.7\text{A}) = 1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}$
$[\vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.3\text{A}) = 7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}]$

(c) $\vec{\tau} = (1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{j}}) = 6.12 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}$
$[\vec{\tau} = (7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{k}}) = -4.68 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}]$

Unit Exam III: Problem #2 (Spring ’14)

(a) Find the magnetic field $\mathbf{B}_a$ (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8$mA [2.7mA]] in the directions shown.
(b) Find the magnetic field $\mathbf{B}_b$ (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5$mA [2.5mA] in the directions shown.



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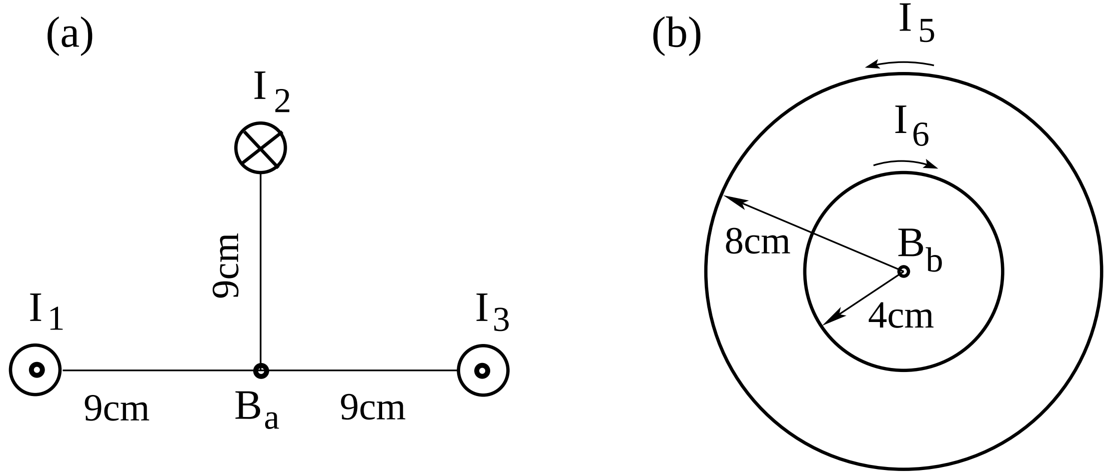


Solution:

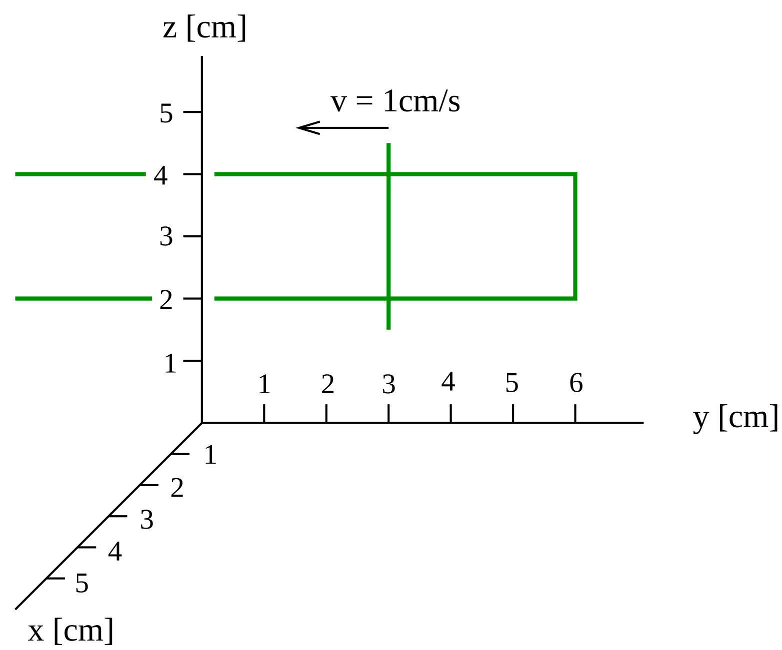
(a) $$B_a = \frac{\mu_0(1.8\text{mA})}{2\pi(9\text{cm})} = 4 \times 10^{-9}\text{T}$$ (directed ←)

$$[B_a = \frac{\mu_0(2.7\text{mA})}{2\pi(9\text{cm})} = 6 \times 10^{-9}\text{T}$$ (directed ←)]

(b) $$B_b = \frac{\mu_0(1.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(1.5\text{mA})}{2(8\text{cm})} = 1.18 \times 10^{-8}\text{T}$$ (directed ⊗)

$$[B_b = \frac{\mu_0(2.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(2.5\text{mA})}{2(8\text{cm})} = 1.96 \times 10^{-8}\text{T}$$ (directed ⊗)]

Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT [$\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT]. A conducting rod slides along conducting rails in the yz-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.
(a) Find the magnetic flux Φ_B through the conducting loop at $t = 0$.
(b) Find the magnetic flux Φ_B through the conducting loop at $t = 1$s.
(c) Find the induced EMF.
(d) Find the direction (cw/ccw) of the induced current.



Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT [$\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT]. A conducting rod slides along conducting rails in the yz-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

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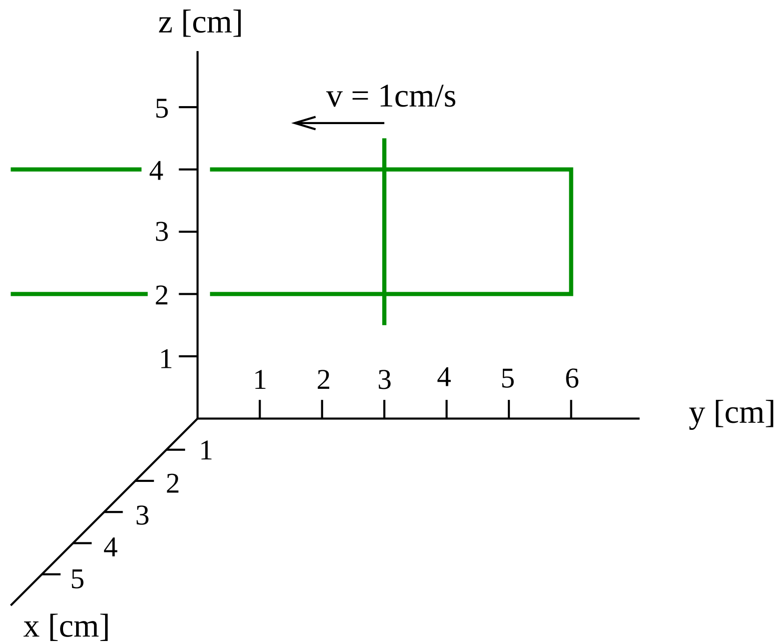

Solution:

(a) $\Phi_B = (3\text{cm})(2\text{cm})(3\text{mT}) = 1.8 \times 10^{-6}\text{Wb}$
$[\Phi_B = (3\text{cm})(2\text{cm})(2\text{mT}) = 1.2 \times 10^{-6}\text{Wb}]$

(b) $\Phi_B = (4\text{cm})(2\text{cm})(3\text{mT}) = 2.4 \times 10^{-6}\text{Wb}$
$[\Phi_B = (4\text{cm})(2\text{cm})(2\text{mT}) = 1.6 \times 10^{-6}\text{Wb}]$

(c) $\mathcal{E} = (1\text{cm/s})(3\text{mT})(2\text{cm}) = 6 \times 10^{-7}\text{V}$
$[\mathcal{E} = (1\text{cm/s})(2\text{mT})(2\text{cm}) = 4 \times 10^{-7}\text{V}]$

(d) cw [cw]