

Oscillator with Two Modes



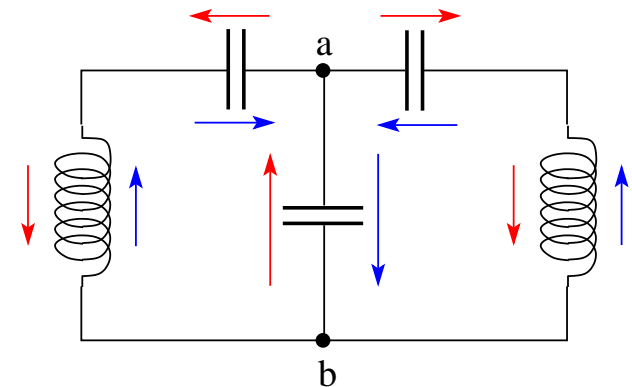
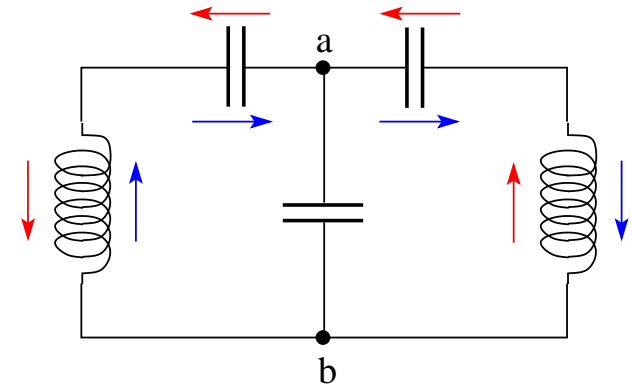
Electromagnetic:

$$\text{mode \#1: } L \frac{dI}{dt} + \frac{Q}{C} + \frac{Q}{C} + L \frac{dI}{dt} = 0, \quad I = \frac{dQ}{dt}$$

$$\Rightarrow \frac{dI}{dt} = -\frac{Q}{LC} \Rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q, \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\text{mode \#2: } L \frac{dI}{dt} + \frac{Q}{C} + \frac{2Q}{C} = 0, \quad I = \frac{dQ}{dt}$$

$$\Rightarrow \frac{dI}{dt} = -\frac{3Q}{LC} \Rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q, \quad \omega = \sqrt{\frac{3}{LC}}$$



Mechanical:

$$\text{mode \#1: } \omega = \sqrt{\frac{k}{m}}$$

$$\text{mode \#2: } \omega = \sqrt{\frac{3k}{m}}$$

