Conservative Forces in Mechanics

Conservative forces familiar from mechanics:

- Elastic force: $F(x) = -kx \quad \Rightarrow U(x) = -\int_{x_0}^{x}(-kx)dx = \frac{1}{2}kx^2 \qquad (x_0 = 0).$
- Gravitational force (locally): $F(y) = -mg$

$$\Rightarrow U(y) = -\int_{y_0}^{y}(-mg)dy = mgy \qquad (y_0 = 0).$$

- Gravitational force (globally): $F(r) = -G\frac{mm_E}{r^2}$

$$\Rightarrow U(r) = -\int_{r_0}^{r}\left(-G\frac{mm_E}{r^2}\right)dr = -G\frac{mm_E}{r} \qquad (r_0 = \infty).$$

Potential energy depends on integration constant.

Integration constant determines reference position where $U = 0$:
$x = x_0,\ y = y_0,\ r = r_0.$