

Work and Potential Energy in 3D Space



Consider a particle acted on by a force \vec{F} as it moves along a specific path in 3D space.

- Force: $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$
- Displacement: $d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$
- Work: $W_{if} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$
- Potential energy: $U(\vec{r}) = - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{s} = - \int_{x_0}^x F_x dx - \int_{y_0}^y F_y dy - \int_{z_0}^z F_z dz$

Note: The work done by a conservative force is path-independent.

