

Potential Energy of Charged Particle in Coulomb Field



- Electrostatic force: $\vec{F} = \frac{kqQ}{r^2} \hat{r}$ (conservative)
- Displacement: $d\vec{s} = d\vec{r} + d\vec{s}_\perp$, $d\vec{r} = dr\hat{r}$
- Work:
$$W_{if} = \int_i^f \vec{F} \cdot d\vec{s} = kqQ \int_i^f \frac{\hat{r} \cdot d\vec{s}}{r^2} = kqQ \int_{r_i}^{r_f} \frac{dr}{r^2}$$
$$= kqQ \left[-\frac{1}{r} \right]_{r_i}^{r_f} = -kqQ \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$$
- Potential energy: $U = - \int_\infty^r F dr = -kqQ \int_\infty^r \frac{dr}{r^2} = k \frac{qQ}{r}$
- Electric potential: $V(r) = \frac{kQ}{r}$

